

MOODY'S KMV RISKCALC™ V3.1 ITALY

MODELING METHODOLOGY

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ABSTRACT

Moody's KMV RiskCalc™ is the Moody's KMV model for predicting private company defaults. It covers over 80% of the world's GDP, has more than 20 geographic specific models, and is used by hundreds of institutions worldwide. While using the same underlying framework, each model reflects the domestic lending, regulator, and accounting practices of its specific region.

In January 2004, Moody's KMV introduced its newest RiskCalc modeling framework, Moody's KMV RiskCalc™ v3.1. By incorporating both market (systematic) and company specific (idiosyncratic) risk factors, RiskCalc v3.1 is in the forefront of modeling middle-market default risk. This modeling approach substantially increases the model's predictive powers.

This document outlines the underlying research, model characteristics, data, and validation results for the RiskCalc v3.1 Italy model.

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1 INTRODUCTION

The Moody's KMV RiskCalc™ v3.1 Italy model is built using the results of extensive Moody's KMV research, including:

- Moody's KMV RiskCalc™ v1.0 and the Moody's KMV Private Firm Model®
- Moody's KMV Credit Research Database™ (CRD, the world's largest and cleanest private company default database)
- Industry sector information, market information, and industry-specific default rates

RiskCalc v3.1 incorporates the structural and market-based comparables approach (used in the MKMV Private Firm Model), and the localized financial statement-based approach (used in RiskCalc v1.0). This allows RiskCalc v3.1 to blend market-based (systematic) information with detailed firm-specific financial statement (idiosyncratic) information to enhance the model's predictive power.

RiskCalc Modes

RiskCalc v3.1 allows a user to assess the risk of a private firm in two ways: Financial Statement Only (FSO) and Credit Cycle Adjusted (CCA).

The Financial Statement Only (FSO) mode delivers a firm's default risk based only on financial statements and sector information, adjusted to reflect differences in credit risk across industries. In this mode, the risk assessments produced by the model are relatively stable over time.

The Credit Cycle Adjusted (CCA) mode adjusts the default risk by taking into account the current stage of the credit cycle. The CCA adjustment is a sector-specific factor derived directly from the Moody's KMV public firm model's distance-to-default. The CCA model reflects the market's current assessment of the credit cycle and is a forward-looking indicator of default.

The CCA adjustment is specific to the firm's sector and country and is updated monthly. The CCA mode also has the ability to stress test EDF credit measures under different credit cycle scenarios – a proposed requirement under Basel II.

RiskCalc v3.1 Italy versus RiskCalc v1.0 Italy

Since the release of RiskCalc v1.0 Italy, Moody's KMV has significantly increased the size of the database for Italy and substantially improved its data cleansing technologies. Furthermore, the new model includes additional financial statement variables, industry adjustments, and a credit cycle adjustment. We have also made substantial advances in our model development and testing techniques. As a result, the new model is more powerful and precise than its predecessor. It also includes additional analytic tools that increase model usability and transparency.

2 DATA DESCRIPTION

The source of the data for RiskCalc v3.1 Italy is Moody's KMV Credit Research Database™ (CRD). Moody's KMV collects data from participating institutions, working closely with them, to understand the strengths and weakness of the data. In countries like Italy, where financial statement data is publicly available, the CRD collects this information as well. As of May 2004, the CRD contained 6.5 million financial statements, over 1.5 million unique private firms, and more than 97,000 default events worldwide. Moody's KMV uses this data for model development and testing purposes.

2.1 Definition of Default

Moody's KMV RiskCalc™ provides assistance to institutions and investors in determining the risk of default, missed payment, or other credit events. The proposals for the new Basel Capital Accord (BIS II) have stimulated debates about what constitutes an appropriate definition of default. RiskCalc applies the criteria used by most of the advanced economies in the world. In model development, RiskCalc uses the local criteria for default, which in Italy is defined as the date of the initiation of insolvency proceedings. At the calibration stages, the model outputs are adjusted to ensure consistent interpretation throughout the world. Specifically, model outputs are converted to a term structure of actual default probabilities (1 through 5-year EDF credit measures).

2.2 Data Exclusions

Excluded Companies

The goal of the RiskCalc model is to provide an Expected Default Frequency™ (EDF) for private Italian companies in the middle market. The firms and industries covered in the model must have similar default characteristics. To create the most powerful model for Italian middle-market companies, companies that did not reflect the typical company in this market were eliminated. The following types of companies are not included in the data:

- **Small companies** – For companies with net sales of less than €500,000 (2002 real Euros), future success is often linked to the finances of the key individuals. For this reason, they are not reflective of typical middle-market companies and are excluded from the database.
- **Financial institutions** – The balance sheets of financial institutions (banks, insurance companies, and investment companies) exhibit higher leverage than the typical private firm. The regulation and capital requirements of these institutions make them dissimilar to the typical middle-market company. Therefore, they are excluded from the database.
- **Real estate development companies** – The annual accounts of real estate development and investment companies provide only a partial description of the dynamics of these firms and, therefore, their likelihood of default. This is because their financial health often hinges on a particular development.¹
- **Public sector and non-profit institutions** – Government run companies' default risks are influenced by the states' or municipalities' unwillingness to allow them to fail. As a result, their financial results are not comparable to other private firms. Not-for-profit financial ratios are very different from for-profit firms, particularly with regard to variables relating to net income.
- **Start-up companies** – Our experience has shown that the financial statements for a company during its first two years are extremely volatile and are a poor reflection of the creditworthiness of the company. The special nature of start-ups is reflected in the fact that many financial institutions have separate credit departments for dealing with these companies.

¹ There are many types of “project finance” firms whose success depends largely on the outcome of a particular project. We recommend using separate models for such firms. At the time of writing, this characteristic is explicitly recognized within the proposals for the new Basel capital accord.

Excluded Financial Statements

The financial statements of smaller companies can be less accurate and of lower quality than those of larger companies. The financial statements in the CRD are cleaned to eliminate highly suspect financial statements. Plausibility checks of financial statements are conducted, such as assets not equal liabilities plus net worth, and financial statements covering a period of less than twelve months. If errors are detected, those statements are excluded from the analysis.

2.3 Descriptive Statistics of the Data

Overview of the Data

The extensive data on both non-defaulting and defaulting companies contained in Moody's CRD has increased substantially since RiskCalc v1.0 was developed. In addition to the increase in time-series data, there has been an increase in the number of participants in the CRD.

Figure 1 presents the distribution of Italian financial statements and defaults by year in the CRD. Table 1 summarizes the data used in the development, validation, and calibration of the RiskCalc v3.1 Italy model.

FIGURE 1 Date Distribution of Italian Financial Statements and Default Data

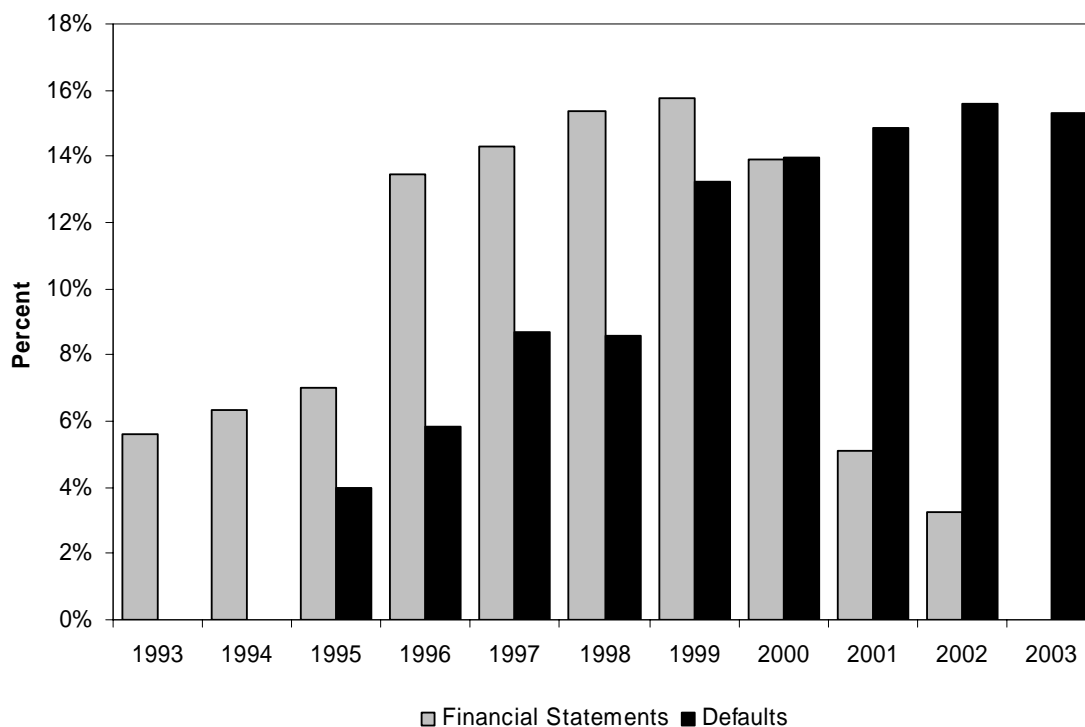


TABLE 1 Information on Italian Private Firm Sample Data

Italian Private Firms	RiskCalc v1.0 Italy	EDF RiskCalc v3.1 Italy	Credit Research Database Growth
Financial statements	124,000+	510,000+	↑311%
Unique number of firms	52,000+	116,000+	↑123%
Defaults	950+	6,600+	↑594%
Time period	1995-1999	1993-2002	↑ 5 additional years

Robustness of the Data

In building a model, potential database weaknesses need to be examined. Not only does the database need to cover a large number of firms and defaults, but the defaults also need to be distributed among industries and company types covered. For example, if the database has significant numbers of small firms or firms in one particular industry and there are not sufficient defaults in those areas, the model may not be a good default predictor. The CRD used in developing the RiskCalc models has addressed both of these issues.

Figure 2 presents the distributions of Italian defaults and firms by industry and the proportion of defaults in each industry. Figure 3 and Figure 4 present similar distributions by the size of firms measured as real total assets and real net sales in 2002 Euros, respectively. These figures demonstrate that the proportion of defaults in any one size group or industry group is comparable to the proportion of firms in these groupings. The size distribution shows that the majority of firms (65%) hold between 1 and 10 million in assets. The proportion of firms between 1 and 10 million in sales is 79% (where firms with less than €500,000 in sales are excluded from the sample).

FIGURE 2 Distribution of Italian Defaults and Firms by Industry

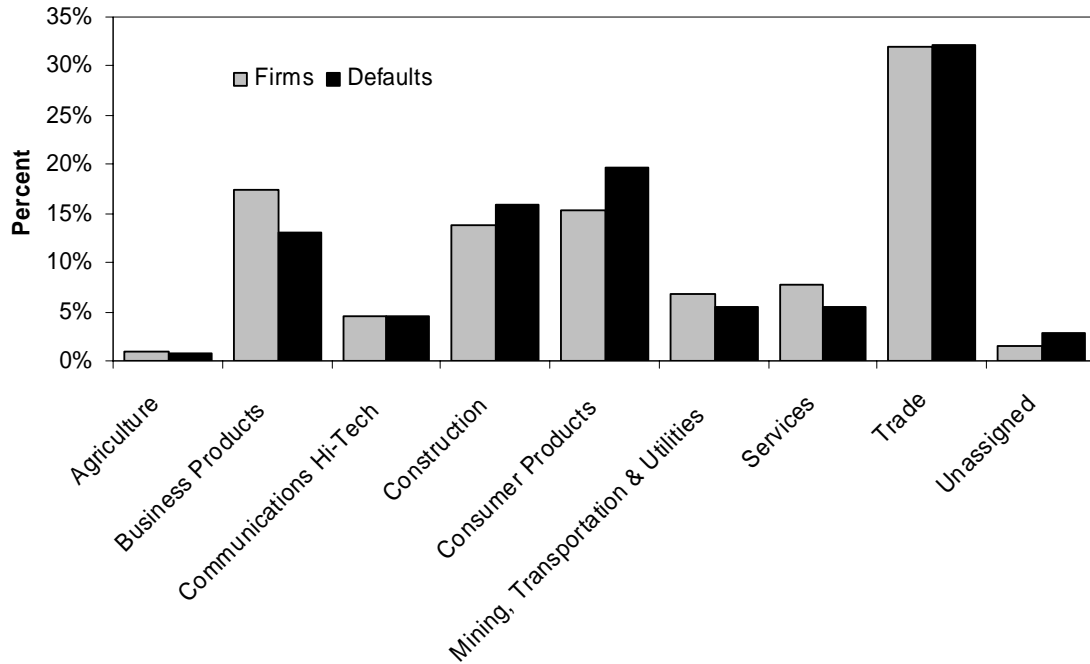


FIGURE 3 Size (as total assets) Distribution of Defaults and Firms

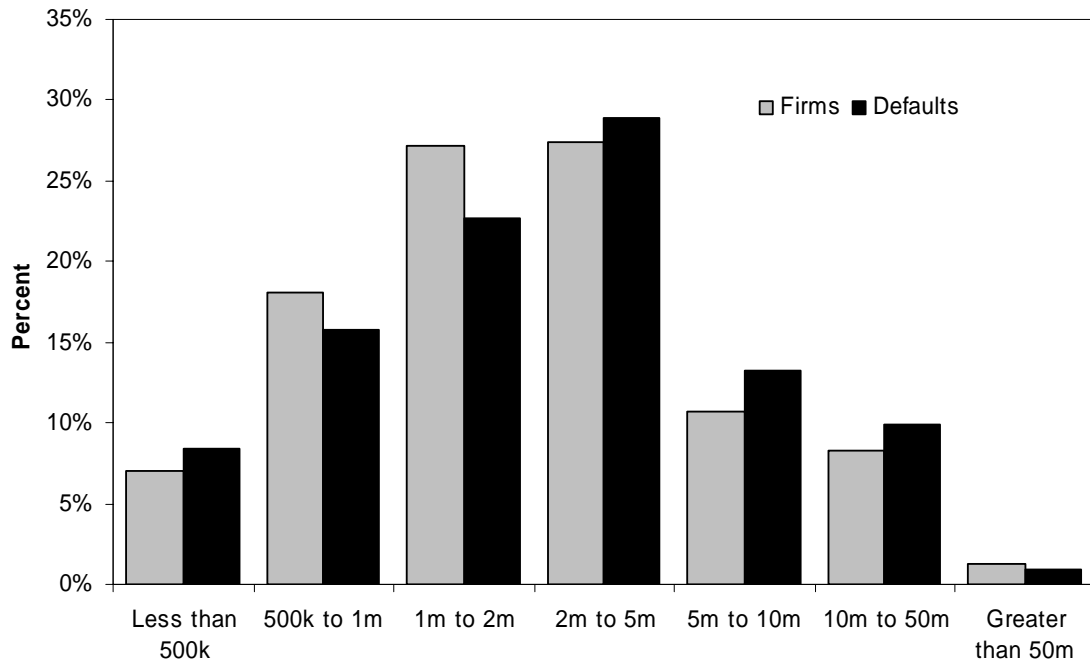
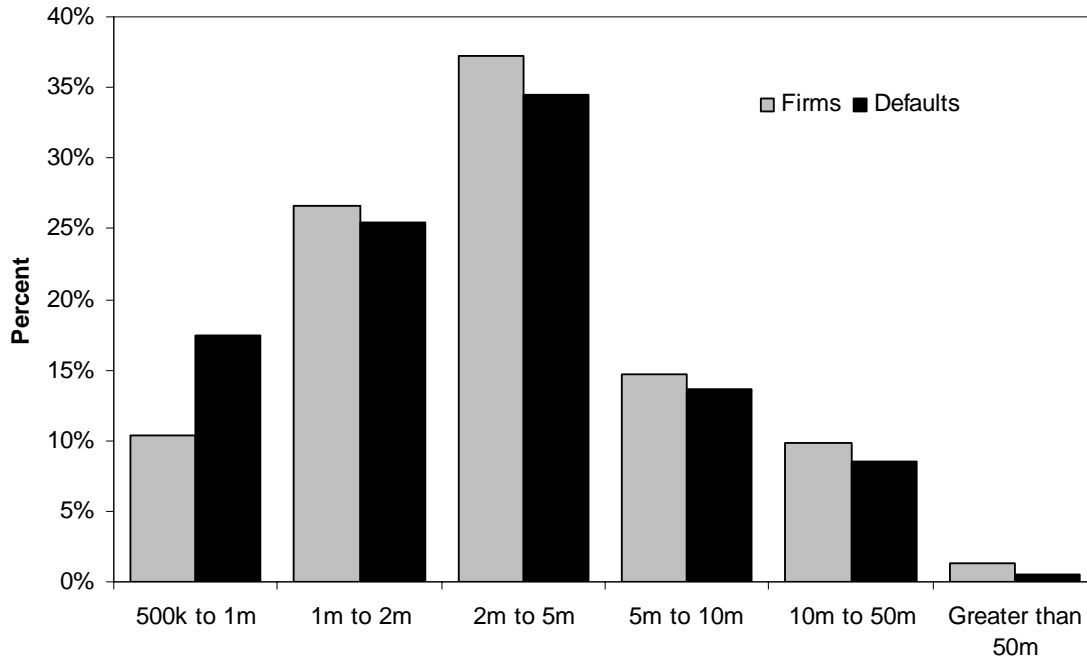


FIGURE 4 Size (as net sales) Distribution of Defaults and Firms



2.4 Cleaning the Data

In the development of a RiskCalc model, the first step is the collection of a large and appropriate database. In addition, data needs to be “cleaned” so that it is representative of the actual risk of the firms covered. MKMV has developed techniques for cleaning the database to improve the model results.

2.5 Central Default Tendency

Since most companies do not default, defaulting companies are relatively rare and thus more valuable in building a default prediction model. Much of the lack in default data is due to the data storage issues within financial institutions such as, defaulting companies being purged from the system after troubles begin, not all defaults being captured, or other sample errors. Also, if the date of default is uncertain the financial statement associated with the firm may be excluded from model development, depending on the severity of the problem. This can result in a sample that has lower default rates than occur in the general population. If the underlying sample is not representative, then it needs to be adjusted for the true central default tendency. When default definitions used in the data sample understate the defaulting population, as is the case with Italy, the central default tendency can be used to realign the default rates.

The estimate of long-run aggregate probabilities of default (or central default tendency) is important as an anchor for a model. The best estimation of default probability is a ratio that would reflect the number of obligors that defaulted in one year compared with the total obligors at the beginning of that year. Given the data problems surrounding the private middle market company data, MKMV consulted prominent Italian financial institutions, as well as Moody’s analysts. Based on these discussions and data review, 2.1% is used as the central tendency figure for the 1-year model.

Calculating a 5-year Central Default Tendency

There is a lack of publicly available data for direct calculation of the central tendency rate of a cumulative 5-year default probability. Based on extensive MKMV research, a 5-year cumulative default tendency is derived from the 1-year estimate. This research, combined with the information provided by the CRD, shows that the 5-year cumulative default rate is, on average, 4 times the level of the 1-year default rate. Therefore, 8.4% is used as the central default tendency for the 5-year model.

Central Default Tendency in FSO and CCA Modes

In the Financial Statement Only model, the central default tendency remains constant over time. In Credit Cycle Adjusted mode, the central default tendency is equal to the central default tendency of the FSO mode when the effects of the credit cycle are neutral. When the forward-looking prediction of the credit cycle indicates increasing default risk, the central default tendency of the CCA mode will be larger, and when the effects of the credit cycle indicate reducing default risk, the central default tendency will be smaller.

3 MODEL COMPONENTS

The RiskCalc v3.1 model incorporates various components to determine the EDF credit measure. The inputs to the model include selection of the financial ratios and transforms of those ratios, the inclusion of industry information, and the credit cycle adjustment.

The development of a RiskCalc model involves the following steps:

1. Choosing a limited number of financial statement variables for the model from a list of possible variables.²
2. Transforming the variables into interim probabilities of default using non-parametric techniques.
3. Estimating the weightings of the financial statement variables, using a probit model, combined with industry variables.
4. Creating a (non-parametric) final transform that converts the probit model score into an actual EDF credit measure.

In FSO mode, the models are based on the following functional form:

$$FSO\ EDF = F \left(\Phi \left(\sum_{i=1}^N \beta_i T_i(x_i) + \sum_{j=1}^K \gamma_j I_j \right) \right)$$

where x_1, \dots, x_N are the input ratios; I_1, \dots, I_K are indicator variables for each of the industry classifications; β and γ are estimated coefficients; Φ is the cumulative normal distribution; F and T_1, \dots, T_N are non-parametric transforms; and FSO EDF is the financial-statement-only EDF credit measure.³ The T s are the transforms of each financial statement variable, which capture the non-linear impacts of financial ratios on the default likelihood. (This is shown in Figure 5 and discussed in detail later in the document.) F is the final transform (i.e. the final mapping). The final transform captures the empirical relationship between the probit model score

² These variables are often ratios but are not always ratios. For example, one measure of profitability is Net Income to Total Assets, which is a ratio, and one measure of size is Inflation Adjusted Total Assets, which is not a ratio.

³ By non-parametric, we mean that the $T(x)$ is a continuous function of x not requiring a specification of a specific closed (or parametric) functional form. We estimate these transforms using a variety of local regression and density estimation techniques.

and actual default probabilities. We describe the final transform as calibrating the model score to an actual EDF credit measure. The difference between the FSO EDF and the credit cycle adjusted EDF is that in CCA mode the final transform is adjusted to reflect our assessment of the current stage of the credit cycle, while in FSO mode it remains constant.

3.1 Financial Statement Variables

Selecting the Variables

Our variable selection process starts with a long list of possible financial statement variables. The working list of ratios is divided into groups that represent different underlying concepts regarding a firm's financial status (see Table 2). A model is then built with at least one variable per group. When it is possible to increase model performance while maintaining model robustness, several variables from each group will be used in the model. Criteria that must be met for variables to be included in the final model are:

- Is the variable readily available?
- Are the definitions of the inputs to the variable unambiguous?
- Is the meaning of the variable intuitive?
- Does the variable predict default activity?
- Is the variable not highly correlated with other variables in the model?

TABLE 2 Groupings of Financial Statement Ratios

Examples of ratios in the **profitability** group include: net income, net income less extraordinary items, EBITDA, EBIT and operating profit in the numerator; and total assets, tangible assets, fixed assets and sales in the denominator. → *High profitability reduces the probability of default.*

Examples of ratios in the **leverage (or gearing)** group include liabilities to assets and long-term debt to assets. → *High leverage increases the probability of default.*

Debt coverage is the ratio of cash flow to interest payments or some other measure of liabilities. → *High debt coverage reduces the probability of default.*

Growth variables are typically the change in ROA and sales growth. These variables measure the stability of a firm's performance. → *Growth variables behave like a double-edged sword: both rapid growth and rapid decline (negative growth) will tend to increase a firm's default probability.*

Liquidity variables include cash and marketable securities to assets, the current ratio, and the quick ratio. These variables measure the extent to which the firm has liquid assets relative to the size of its liabilities. → *High liquidity reduces the probability of default.*

Activity ratios include inventories to sales and accounts receivable to sales. These ratios may measure the extent to which a firm has a substantial portion of assets in accounts that may be of subjective value. For example, a firm with a lot of inventories may not be selling its products and may have to write off these inventories. → *A large stock of inventories relative to sales increases the probability of default; other activity ratios have different relationships to default.*

Size variables include sales and total assets. These variables are converted into a common currency as necessary and then are deflated to a specific base year to ensure comparability (e.g., total assets are measured in 2002 Euros). → *Large firms default less often.*

TABLE 3 Financial Statement Variables used in RiskCalc v3.1 Italy*

Category	Variable
Activity	Change in Current Receivables Turnover Financial Charges to Sales
Debt Coverage	Ordinary Profit + Depreciation & Amortization to Financial Charges
Growth	Sales Growth
Leverage	(Net Worth – Intangible Assets) to (Assets – Intangible Assets)
Liquidity	Current Assets to Current Liabilities
Profitability	(Net Income + Taxes) to Assets
Other	Cash to Current Assets
Size	Total Assets

*Current Receivables Turnover is the ratio of receivables due within one year to sales.

Variable Transforms

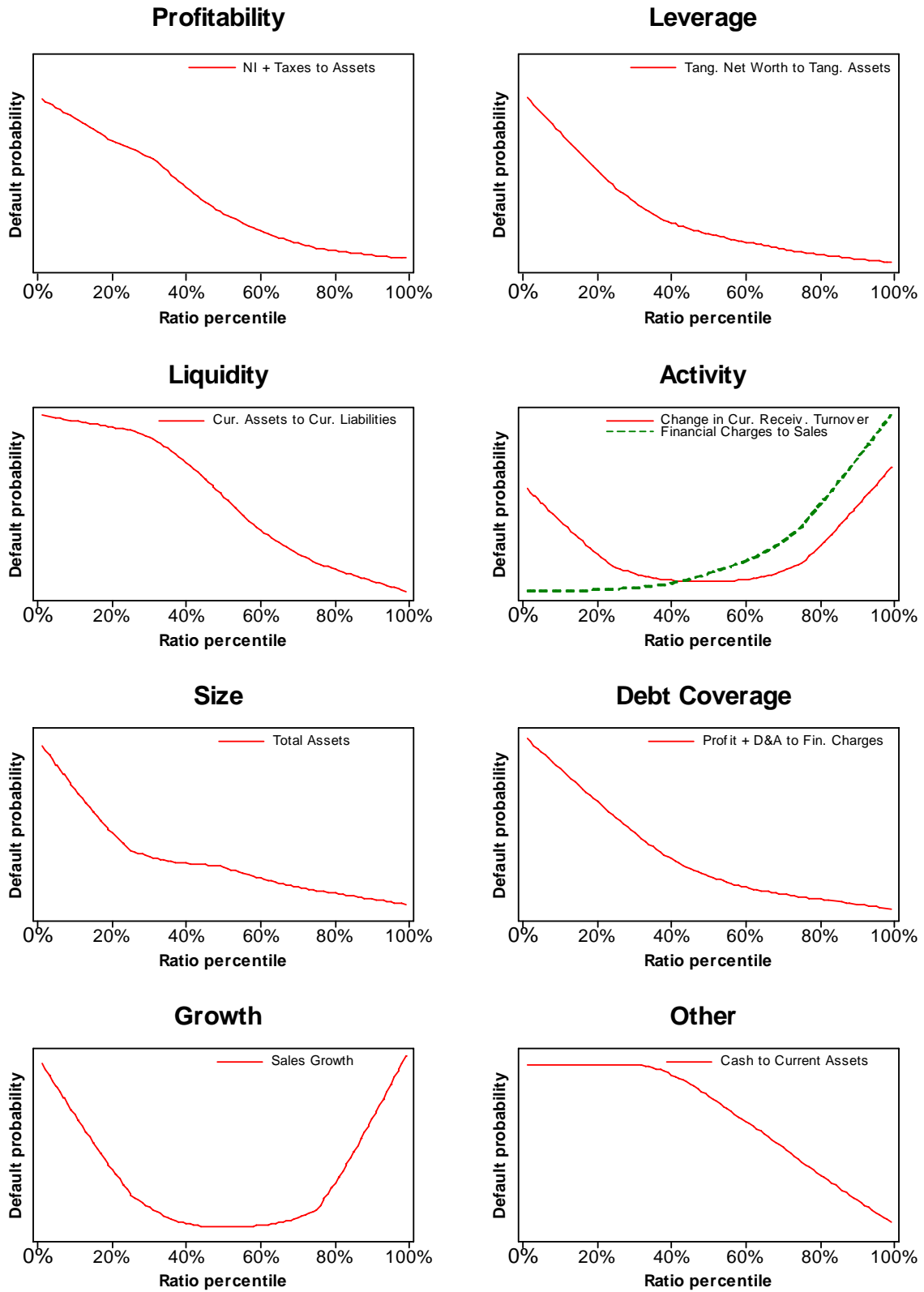
Once the variables are selected, they are transformed into a preliminary EDF value. Figure 5 presents the transformations used in the model. The horizontal axis gives the percentile score of the ratio and the vertical axis gives the default probability of that ratio in isolation (univariate). The percentile score represents the percent of the database that had a ratio below that of the company (e.g., if ROA is in the 90th percentile that means that 90% of the sample had an ROA lower than that firm).

The shape of the transformation indicates how significantly a change in level impacts the EDF value. If the slope of the transform is steep, a small change will have a larger impact on risk than if the slope were flat.

- For the **Profitability** group, the transform for NI + Taxes to Assets is downward sloping, but the slope becomes almost zero as profitability becomes large (Figure 5). Such a transform indicates that more profitable firms have lower default probabilities, but the impact diminishes as profitability increases.
- For the **Leverage** group, the transform for Tangible Net Worth to Tangible Assets is downward sloping since leverage increases as the ratio decreases. Large leverage corresponds to low levels of Tangible Net Worth to Tangible Assets and high default risk. The slope becomes less negative as the ratio increases, which implies that a small increase in leverage, when leverage is high (and equity is low), will increase the default likelihood by a larger amount than when leverage is low (and equity is high) (Figure 5).
- For the **Liquidity** group, the transform for Current Assets to Current Liabilities is downward sloping and backwards “S shaped”. The largest changes in default probability occur when moving upward or downward from the median of the distribution (Figure 5).
- For the **Activity** group, two ratios are included. Financial Charges to Sales are upward sloping indicating that high values of these ratios are associated with higher default probabilities. However, the slope is almost zero until the 30th percentile of the ratio and begins to increase rapidly after that point (Figure 5). Change in Current Receivables Turnover is “U shaped,” indicating that large positive values or large negative values are associated with higher default probabilities, while stable current receivables turnover is associated with lower default probabilities.
- The **Size** variable is Total Assets. This variable's transformation is downward sloping, but the slope becomes almost zero as size becomes large (Figure 5). This indicates that larger firms have lower default probabilities, but the impact of size on default probabilities is diminishing as firm size increases.

- The **Debt Coverage** variable is Ordinary Profit + Depreciation & Amortization to Financial Charges. This variable is downward sloping indicating that large values of cash flow relative to interest expense lower the probability of default. The slope becomes almost zero as debt coverage increases (Figure 5).
- The **Growth** variable is Sales Growth. It is “U shaped,” indicating that large increases or decreases in sales are associated with higher default probabilities, while stable sales year upon year decreases the probability of default (Figure 5).
- The **Other** variable is Cash to Current Assets, which measures the “quality” of current assets. The slope is zero for high values of the ratio and becomes downward sloping as the ratio increases (Figure 5). This indicates that until Cash dips below a certain percentage level of Current Assets the probability of default is unaffected.

FIGURE 5 Transformations of Financial Statement Variables Used in the Model



3.2 Model Weights

Importance

The relative value of each variable used in calculating an EDF credit measure is important in understanding a company's risk. The non-linear nature of the model makes the weight of the variables more difficult to determine since the actual impact on the risk depends on the coefficient, the transformation shape, and the percentile ranking of the company. The model weights, therefore, are calculated based on the average EDF value for the transformation and its standard deviation. A variable with a flat transformation could have a low weight, even if the coefficient is large (Figure 5).

Calculation of Weights

To calculate the weighting of a variable, the EDF credit measure for a theoretical firm with all its variables at the average transformation values is computed. The variables are then increased one at a time by one standard deviation. The EDF change for each variable (in absolute value) is computed and added together. The relative weight of each variable is then calculated as the EDF level changes for that variable as a percent of the total change in EDF level. This gives the variable with the biggest impact on the EDF level the biggest weight, and the variable that has the smallest impact on the EDF level the smallest weight. Since the weights are a percentage of the total EDF value, they sum to 100%. The weight of each category is the sum of the weights of each variable in that category. Table 4 presents the weights in RiskCalc v3.1 Italy.

TABLE 4 Risk Drivers in RiskCalc v3.1 Italy

Risk Drivers	Weight
Profitability (Net Income + Taxes)/Assets	11%
Leverage Tangible Net Worth to Tangible Assets	26%
Debt Coverage Ordinary Profit + Depreciation & Amortization/ Financial Charges	19%
Activity Financial Charges/Sales Change in Current Receivables Turnover	19%
Growth Sales Growth	6%
Other Cash/Current Assets	8%
Liquidity Current Assets/Current Liabilities	5%
Size Total Assets	6%

3.3 Industry Adjustments

While the variables included in the RiskCalc model explain most of the risk factors, the relative importance of the variables can be different among industries. In the FSO mode of RiskCalc v3.1 Italy, the EDF value is adjusted for industry effects. Table 5 presents the increase in model power and accuracy from introducing industry controls into the FSO model. Both the power and the accuracy of the EDF credit measure increase, as measured by the accuracy ratio and the gain in log likelihood. The large gain in likelihood indicates that the industry controls are especially important in producing an accurate EDF. Table 6 presents the average EDF value by industry for the development sample in April of 2003.

TABLE 5 Increase in Model Power and Accuracy from Introducing Industry Controls

	One-year Model		Five-year Model	
	Accuracy Ratio	Relative increase in Log Likelihood	Accuracy Ratio	Relative increase in Log Likelihood
FSO mode without industry controls	73.9%		59.0%	
FSO mode with industry controls	74.2%	203.1***	59.7%	492.4***

*** Indicates a P-value of less than 0.01 percent.

In this table, and hereafter, Accuracy Ratio (or AR) is the measure of the model's ability to rank order credits. Increases in log likelihood measure the extent to which the model's EDF values match observed default rates. For further details, see Dwyer and Stein (2004), *Technical Document on RiskCalc v3.1 Methodology* (Technical Document).

TABLE 6 Average EDF Credit Measure in April 2003 by Sector

Sector	Average 1-year EDF	Average 5-year EDF
Agriculture	1.96%	7.58%
Business Products	1.82%	6.01%
Communications and Hi Tech	2.49%	7.82%
Construction	1.87%	6.93%
Consumer Products	3.18%	10.90%
Mining, Transportation, Utilities and Natural Resources	1.49%	5.49%
Services	1.79%	5.96%
Trade	2.37%	7.68%

3.4 Credit Cycle Adjustment

EDF credit measures are impacted not only by the financials of a company, but also by the general credit cycle in the economy. To capture this effect, RiskCalc v3.1 Italy includes a credit cycle adjustment factor. The credit cycle adjustment is designed to incorporate the current credit cycle into the estimate of private firm default risk.

Selecting an Adjustment Factor

The RiskCalc v3.1 model uses the distance-to-default calculation from the Moody's KMV public firm model. This measure is specifically designed to be a forward-looking indicator of default risk. It extracts signals of default risk from the stock market performance of individual firms (cf., Bohn and Crosbie, 2003). This measure was chosen because it is available for a large universe of industries and it has been extensively validated.

If the distance-to-default for public firms in an industry indicates a level of risk above the historical average for that industry, then the private firms' EDF values in that industry are adjusted upward. Conversely, if the level of risk is below the historical average, then the private firms' EDF values are adjusted downward. When the credit cycle adjustment factor is neutral, the CCA EDF value coincides with the FSO EDF value.

Adjustment Factor used in the Model

For the Italian model, the distance-to-default factor for each industry is a weighted average of two indices. The average is based on the aggregation of distance-to-default for all Italian and continental European firms in each industry⁴. The weight on the Italian factor is industry specific and determined by the value (assets) of Italian firms in each industry relative to all of continental Europe. In the event that a firm cannot be associated with a specific industry, the model uses a credit cycle adjustment based on an aggregation of all public Italian and continental European firms.

Figure 6 presents the evidence of the Italian distance-to-default factor and insolvency levels in Italy. The distance-to-default factor is inverted when graphed so that higher values on the graph indicate higher probabilities of default for Italian public firms. The distance-to-default tracks insolvency levels in a *forward-looking* manner throughout the period. Data on insolvency levels is compiled by Creditreform (2004).

Figure 7 provides evidence of the relationship between the distance-to-default factor and public default rates in continental Europe as measured by Moody's KMV.⁵ Similar to private firm defaults, the factor is a *forward-looking* measure of the probability of default for public European firms. Overall, the private and public default evidence shows that the distance-to-default factor is a strong predictor of economic conditions in each industry and will adjust the probabilities default to reflect the position in the credit cycle. Table 7 shows that including the credit cycle adjustment factor increases both the power and the accuracy of the model.

⁴ Continental European firms used to compute the distance-to-default factor are as follows: Austria, Belgium, Denmark, France, Germany, Greece, Italy, Luxembourg, the Netherlands, Portugal, Spain, and Switzerland.

⁵In this context, a public company is a company with publicly traded equity. There are too few public firm defaults in Italy to construct a meaningful default rate at this time.

FIGURE 6 Italian DD Factor and Insolvencies in Italy: 1997-2003

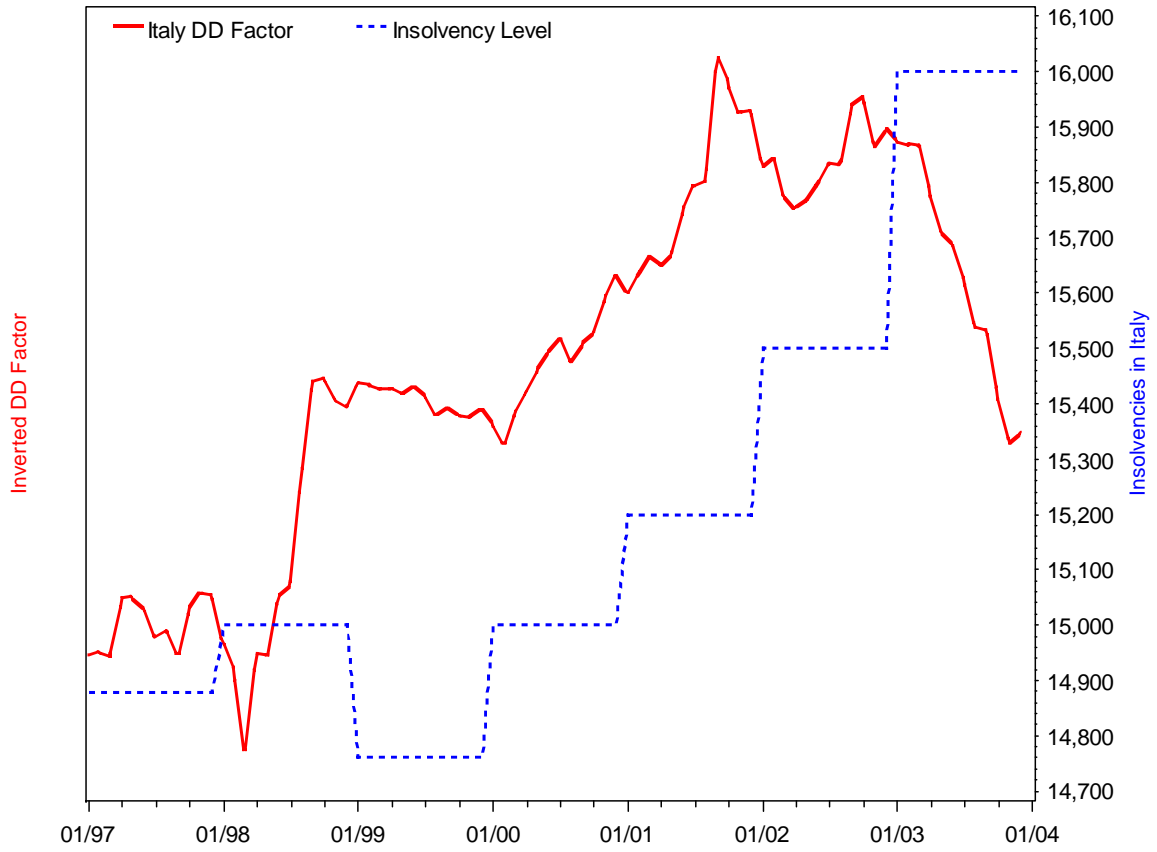


Figure 6 gives the DD factor (red line) against the historical insolvencies levels in Italy (blue line). The DD factor increases in anticipation of the increase in default activity. Insolvency data is compiled by Creditreform (2004).

FIGURE 7 Continental Europe DD Factor and Public Default Rates: 1997-2003

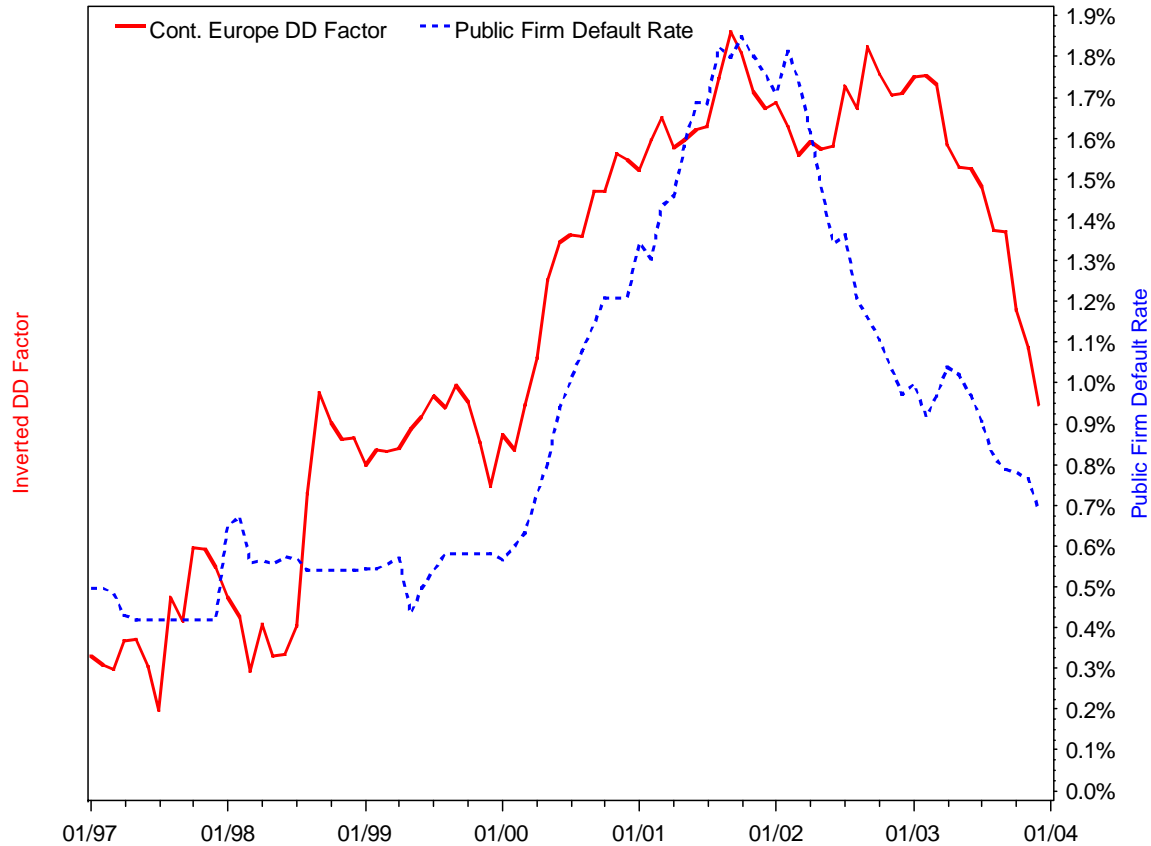


Figure 7 gives the DD factor (red line) against the historical public bond default rate for continental Europe (blue line). The DD factor increases in anticipation of the increase in default activity.

4 VALIDATION RESULTS

Once a model is developed, it must be shown to be effective in predicting defaults. In this section, we present testing results on the model's ranking power (the model's ability to sort credits from worst to best) and the accuracy of its predicted EDF credit measure (the model's ability to estimate the level of EDF correctly).

The tests need to check not only the model effectiveness, but also its robustness and how well it works on data outside the sample. To do out-of-sample testing, we performed walk-forward and *k*-fold analyses. The results of the testing show that the model is uniformly more powerful than other models across different time periods, sectors, and size classifications.

4.1 Increase in Overall Model Power and Accuracy

Table 7 presents the in-sample overall measures of power and likelihood for RiskCalc v3.1 Italy versus alternative models. With the credit cycle adjustment, the model's performance improves by almost three percentage points of accuracy ratio at the 1-year horizon and over four percentage points at the 5-year horizon compared with RiskCalc v1.0 Italy. Relative to other available alternatives, the results were more dramatic. The new RiskCalc v3.1 model outperformed the Z-score model (Altman, Hartzell and Peck, 1995) by more than twenty percentage points at both the 1-year and 5-year horizons. The Financial Statement Only (FSO) mode

outperforms the old model by two percentage points at the 1-year horizon and four percentage points at the 5-year horizon. RiskCalc v3.1 Italy is also more accurate than alternative models as measured by the log-likelihood differences.⁶

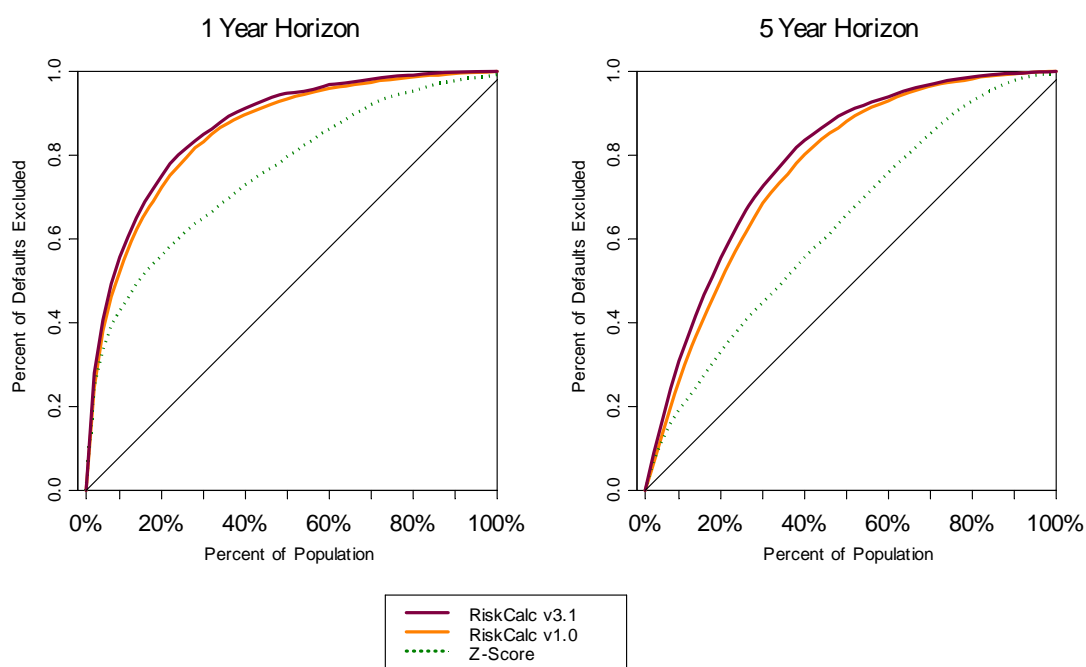
TABLE 7 Power Enhancements of the new RiskCalc v3.1 Italy Model

	One-year Model		Five-year Model	
	Accuracy Ratio	Lead in Log Likelihood*	Accuracy Ratio	Lead in Log Likelihood*
RiskCalc v3.1 Model	74.5%		59.7%	
RiskCalc v3.1 Model - FSO	73.9%	106	59.6%	420
RiskCalc v1.0	71.8%	990	55.2%	985
Z-score	52.8%	9725	28.7%	4915

*Presents the increase in log likelihood of RiskCalc v3.1 Italy over the alternative model. Larger values indicate that levels of RiskCalc v3.1 are better-calibrated vis-à-vis the alternative model.

Figure 8 presents the cumulative accuracy profiles for the 1- and 5-year models corresponding to Table 7. The power improvements are largely in the middle of the distribution relative to RiskCalc v1.0 (particularly for the 1-year model). This result implies that both very good and very poor credits are correctly identified by both RiskCalc v1.0 and RiskCalc v3.1. The added discriminatory power is assessing the credit quality of credits that fall in the middle range.

FIGURE 8 Power of Alternative Models (1- and 5-year) — Italy



⁶ The log likelihood can be thought of as a measure of closeness of the predicted EDF values to the actual default rates.

4.2 Correlations and Variance Inflation Factors

To ensure model robustness, the model must be tested for excessive multicollinearity, which occurs if a number of the variables used in the model are highly correlated. Excessive multicollinearity can cause instability in parameter estimates. In order to check for this issue, the correlation coefficients (Table 8) for the financial statement ratios in the model and the variance inflation factors (Table 9) are computed on the transformed variables (see Figure 5).⁷

Model Results

The highest correlation coefficient is between [Ordinary Profit + Depreciation & Amortization to Financial Charges] and [Net Income + Taxes to Total Assets] (0.73). The next highest coefficient is between [Ordinary Profit + Depreciation & Amortization to Financial Charges] and [Financial Charges to Sales] (0.64). Such coefficients are below what we would typically consider indications of multicollinearity, and this finding is also verified by the VIF analysis.

TABLE 8 Correlations Among the Transformed Input Factors (Spearman Rank)

	Ordinary Profit + Depreciation & Amortization to Financial Charges	(Net Income + Taxes) to Assets	Financial Charges to Sales	Tangible Net Worth to Tangible Assets	Current Assets to Current Liabilities	Sales Growth	Total Assets	Change in Current Receivables Turnover	Cash to Current Assets
Ordinary Profit + Depreciation & Amortization to Financial Charges	1.0								
(Net Income + Taxes) to Assets	0.734	1.0							
Financial Charges to Sales	0.636	0.419	1.0						
Tangible Net Worth to Tangible Assets	0.439	0.456	0.244	1.0					
Current Assets to Current Liabilities	0.314	0.360	0.310	0.602	1.0				
Sales Growth	0.145	0.131	0.106	0.112	0.069	1.0			
Total Assets	-0.028	-0.061	-0.255	0.163	0.076	0.036	1.0		
Change in Current Receivables Turnover	0.104	0.100	0.177	0.063	0.031	0.294	-0.053	1.0	
Cash to Current Assets	0.289	0.257	0.338	0.186	0.171	0.041	-0.116	0.077	1.0

⁷ For further definitions and technical discussion of the testing procedures in Section 4 refer to the Technical Document.

The Variance Inflation Factors (Table 9) for the financial statement variables represent how much of the variation in one independent variable can be explained by all the other independent variables in the model. The correlation coefficient, however, measures only the relationships between two variables. As shown in Table 9, the estimated VIF values in the Italy model are notably below the threshold levels of 4 to 10 that are commonly used in VIF analysis when testing for presence of multicollinearity.⁸ These findings indicate that the model variables do not present any substantial multicollinearity.

Further tests were performed to ensure that multicollinearity is not a problem in the Italian model. Tests for the usual effect of multicollinearity, which is model parameter instability, show that the model parameters are stable across random sub-samples. In other words, re-estimating the model on small random sub-samples of the data does not produce economically meaningful differences in the model's coefficients.

TABLE 9 Variance Inflation Factors

Variable	VIF
Ordinary Profit + Depreciation & Amortization to Financial Charges	2.7
[Net Income + Taxes] to Assets	2.3
Financial Charges to Sales	1.7
[Net Worth - Intangible Assets] to [Assets - Intangible Assets]	1.7
Current Assets to Current Liabilities	1.5
Sales Growth	1.2
Total Assets	1.2
Change in Current Receivables Turnover	1.2
Cash to Current Assets	1.1

4.3 Model Power by Industry and Size Groups

It is important to test the power of a model not only overall, but also among different industry segments and firm sizes. Table 10 and Table 11 present the power comparisons by sector for the 1-year and 5-year models, respectively.

RiskCalc v3.1 Italy outperforms both RiskCalc v1.0 Italy and Z-score in all sectors. The highest power in the 1-year model is found in Business Products (81.1%) while the lowest is found in Agriculture (62.2%). At the 5-year horizon (Table 11) the highest power is in Business Products (66.0%) and the lowest is in Trade (55.7%).

⁸ As Woolridge (2000) shows VIF is inversely related to the tolerance value ($1-R^2$), such that a VIF of 10 corresponds to a tolerance value of 0.10. Clearly, any threshold is somewhat arbitrary and depends on the sample size. Nevertheless, if any of the R^2 values are greater than 0.75 (so that VIF is greater than 4.0), we would typically suspect that multicollinearity could be a problem. If any of the R^2 values are greater than 0.90 (so that VIF is greater than 10) we then conclude that multicollinearity is likely to be a serious problem.

TABLE 10 Model Power by Industry 1-year Model

	Percentage of Defaults*	AR RiskCalc v3.1	AR RiskCalc v1.0	AR Z-score
Agriculture	0.6%	62.2%	56.4%	44.4%
Business Products	13.6%	81.1%	80.3%	65.2%
Communications and Hi Tech	4.7%	79.8%	77.9%	65.2%
Construction	17.1%	70.1%	66.5%	47.5%
Consumer Products	21.8%	77.1%	74.5%	54.4%
Mining, Transportation, Utilities and Natural Resources	5.3%	79.6%	76.6%	54.2%
Services	4.5%	70.1%	68.7%	54.4%
Trade	32.3%	69.7%	65.6%	46.8%

* The total does not sum to 100% due to rounding

TABLE 11 Model Power by Industry 5-year Model

	Percentage of Defaults*	AR RiskCalc v3.1	AR RiskCalc v1.0	AR Z-score
Agriculture	0.7%	58.6%	46.5%	22.4%
Business Products	13.3%	66.0%	64.0%	40.4%
Communications and Hi Tech	4.7%	62.3%	58.1%	31.3%
Construction	16.6%	54.9%	49.4%	21.5%
Consumer Products	21.0%	59.2%	55.8%	25.5%
Mining, Transportation, Utilities and Natural Resources	5.7%	63.6%	59.4%	26.2%
Services	5.2%	58.2%	54.2%	34.0%
Trade	32.7%	55.7%	50.4%	27.0%

* The total does not sum to 100% due to rounding

Table 12 and Table 13 present the power comparisons by firm size for the 1-year and 5-year models, respectively. RiskCalc v3.1 Italy out performs both RiskCalc v1.0 Italy and Z-score in all size groups. The highest power in the 1-year model is found in the largest firms—over €50 million in assets, and the lowest is in the smallest firms—under €500,000 in assets. A similar relationship is often found between model power and size in other countries. Such performance improvements are likely to reflect the higher quality of financial statements among larger firms.

TABLE 12 Model Power by Size 1-year model

	Percentage of Defaults*	AR EDF RiskCalc v3.1	AR RiskCalc v1.0	AR Z-score
<€500,000	6.0%	49.6%	49.2%	39.5%
€500,000 to €1mm	12.4%	64.5%	62.9%	44.3%
€1mm to €2mm	22.9%	75.0%	73.6%	51.4%
€2mm to €5mm	31.0%	76.5%	73.4%	51.3%
€5mm to €10mm	15.1%	77.4%	75.4%	55.9%
€10mm to €50mm	11.6%	77.9%	76.8%	55.9%
over €50mm	1.2%	78.9%	78.9%	62.5%

* The total does not sum to 100% due to rounding

TABLE 13 Model Power by Size 5-year model

	Percentage of Defaults	AR RiskCalc v3.1	AR RiskCalc v1.0	AR Z-score
<€500,000	6.4%	38.9%	36.6%	21.3%
€500,000 to €1mm	12.2%	51.8%	48.7%	26.2%
€1mm to €2mm	22.9%	63.4%	59.6%	31.4%
€2mm to €5mm	31.1%	64.1%	60.8%	33.2%
€5mm to €10mm	15.3%	63.0%	61.2%	35.9%
€10mm to €50mm	10.9%	64.0%	61.8%	33.7%
over €50mm	1.2%	71.4%	69.7%	38.1%

4.4 Power Performance Over Time

Since models are implemented at various points in a business cycle by design, power tests by year (Table 14 and Table 15) were conducted to examine whether or not the model performance is excessively time dependent.

Table 14 and Table 15 present the results from this analysis at the 1- and 5-year horizons, respectively. The AR of RiskCalc v3.1 Italy is compared with RiskCalc v1.0 Italy and Z-score for each year. As shown in these tables, RiskCalc v3.1 consistently outperforms both by a considerable margin.

TABLE 14 Model Power over Time: 1-year Horizon

	Percent of Defaults	AR* RiskCalc v3.1	AR RiskCalc v1.0	AR Z-score
1993	6.8%	68.9%	67.4%	39.8%
1994	7.4%	71.2%	70.0%	49.1%
1995	8.0%	75.6%	75.2%	53.4%
1996	14.3%	73.0%	71.1%	48.7%
1997	16.7%	73.9%	70.9%	48.0%
1998	17.5%	74.3%	72.2%	49.1%
1999	16.9%	71.4%	68.9%	47.7%
2000	12.4%	67.8%	65.7%	41.9%

*AR = accuracy ratio

TABLE 15 Model Power over Time: 5-year Horizon

	Percent of Defaults	AR* RiskCalc v3.1	AR RiskCalc v1.0	AR Z-score
1993	9.0%	60.6%	58.4%	26.4%
1994	8.9%	62.7%	60.6%	32.8%
1995	9.8%	64.6%	62.5%	37.8%
1996	18.8%	63.7%	60.3%	35.0%
1997	20.3%	64.0%	60.0%	34.5%
1998	19.0%	65.7%	63.2%	39.2%
1999	14.2%	67.3%	64.7%	41.8%

*AR = accuracy ratio

4.5 Out of Sample Testing: k -fold Tests

The model exhibits a high degree of power in distinguishing good credits from bad ones (in Table 7), but whether this power is attributable to the overall model effectiveness or the impact of a particular sub-sample also needs to be tested. A standard test for evaluating this is the “ k -fold test,” which divides the defaulting and non-defaulting companies into k equally sized segments. This yields k equally sized observation sub-samples that exhibit the identical overall default rate and are temporally and cross-sectionally independent. The model is then run on $k-1$ sub-samples and these parameter estimates are used to score the k -th sub-sample. We repeat this procedure for all possible combinations, and put the k scored “out-of-sample” sub-samples together to calculate an accuracy ratio on this combined data set.

Table 16 summarizes the k -fold test results (with $k=5$). The reported figures are the accuracy ratios by the corresponding sample and time horizons. The out-of-sample model consistently outperforms RiskCalc v1.0 Italy. Figure 9 presents the cumulative accuracy profiles associated with the overall “out-of-sample” results against the in-sample results. The model performance is maintained both in- and out-of-sample in the k -fold analysis.

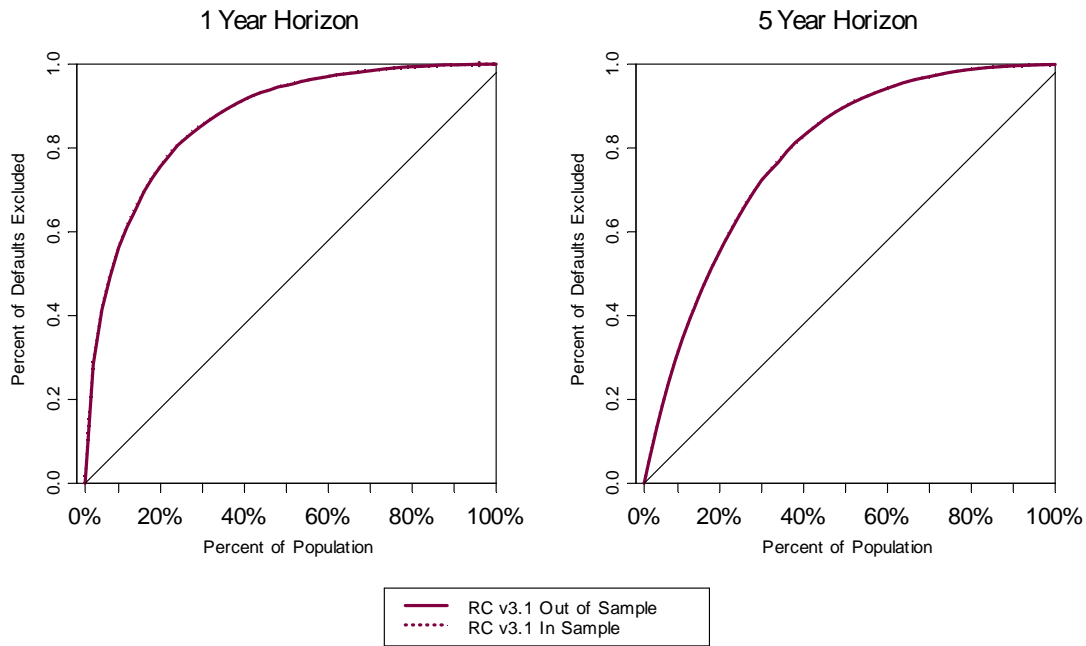
Results

The in- and out-of-sample plots are virtually indistinguishable at both the 1- and 5-year horizons in Figure 9. The difference in AR between the overall in-sample and out-of-sample results is not larger than 20 basis points in both cases. Furthermore, RiskCalc v3.1 Italy outperforms RiskCalc v1.0 Italy in an out-of-sample context at both the 1- and 5-year horizons (Table 16).

TABLE 16 RiskCalc v3.1 Italy k -fold Test Results

	Out of Sample AR		RiskCalc v1.0	
	1-year AR	5-year AR	1-year AR	5-year AR
Subsample 1	72.2%	62.6%	70.4%	59.2%
Subsample 2	72.7%	62.6%	70.5%	58.9%
Subsample 3	72.9%	64.6%	70.0%	60.4%
Subsample 4	73.5%	63.7%	70.8%	60.1%
Subsample 5	73.3%	63.8%	70.8%	60.0%
K-fold Overall	75.1%	59.1%	72.5%	53.3%
In-sample AR	75.3%	59.2%		

FIGURE 9 RiskCalc v3.1 Italy k -fold



The k -fold testing does not control for time dependence. Each of the k sub-samples contains data from all periods. As a result, if there were a particularly high period of default rates, this would be included in each of the k samples. Such testing does not give a true sense of the how the model would have performed during those volatile periods because the model is estimated with full information on those time periods.

4.6 Walk-Forward Tests

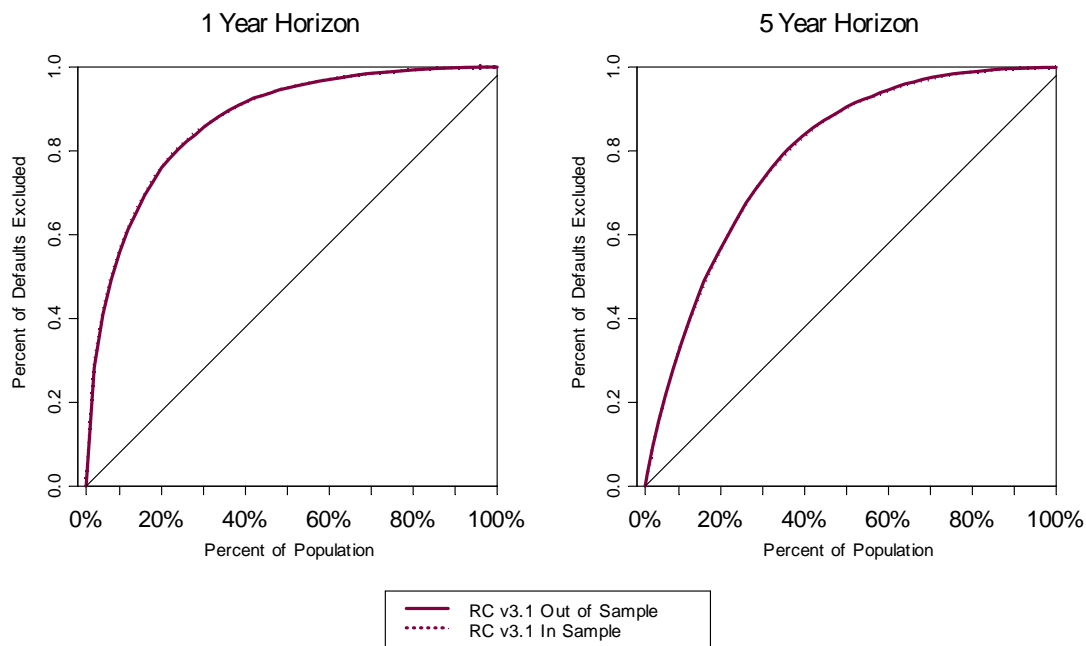
An alternative out-of-sample test developed by Moody's KMV is a *walk-forward* analysis, which is designed along similar lines as the k -fold test, except that it controls for the effects of time. The model is estimated up to a certain year and the parameter estimates are then used to score the observations in the *next* year. These model scores are *out-of-time*. The model is re-estimated including one more year of data and we repeat the analysis for the next year and continue until the end of the sample. These out-of-sample out-of-time scores are combined into a single prediction set so that the accuracy ratio and the power curve can be calculated for the combined set. The out-of-sample accuracy ratio is then compared to the corresponding in-sample accuracy ratio and power curve.

No data from a future period is used in fitting the model and only data from future periods is used for testing it. The parameter estimates are checked for stability across the different samples. Figure 10 presents the results from this analysis.

Results

Similar to the k -fold results, the in- and out-of-sample plots for the walk-forward results are virtually indistinguishable at both the 1- and 5-year horizons in Figure 10. The difference in AR between the in-sample and out-of-sample results is no more than 30 basis points in both cases. Furthermore, RiskCalc v3.1 Italy outperforms RiskCalc v1.0 Italy in an out-of-time context at both the 1- and 5-year horizons.⁹

FIGURE 10 Out-of-sample Performance (1- and 5-year) Italy Walk-forward



4.7 Model Calibration and Implied Ratings

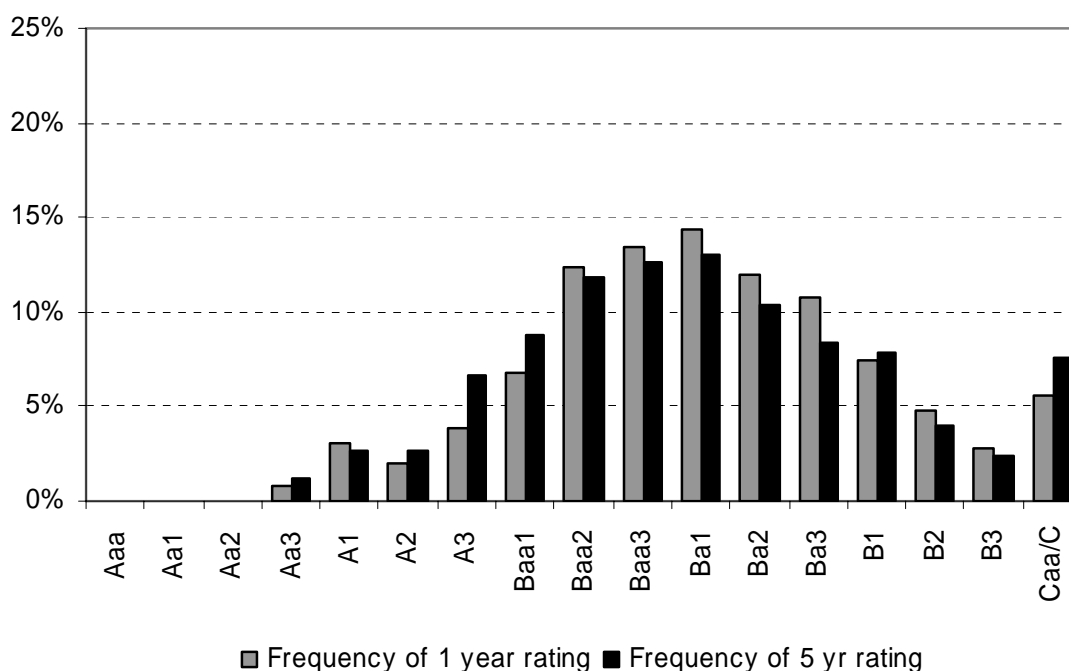
To aid in the interpretation of an EDF credit measure, an EDF value is mapped to an .edf rating (an EDF-implied rating). All RiskCalc v3.1 models to date have used the same mapping. This mapping is designed so that:

- There is a large range of .edf ratings (as required for economic and regulatory applications);
- No one rating contains too many credits (as required for economic and regulatory applications);
- The distribution of the 5-year ratings is approximately the same as the distribution of 1-year ratings (for consistency with rating-based analysis applications);
- The EDF value associated with an .edf rating is approximately the same as the observed historical default rate associated with a Moody's Bond Rating (for consistency with rating-based analysis applications).

⁹ The out-of-sample ARs are 75.2% and 60.5% for the 1-year and 5-year models, respectively. The 1-year out-of-sample power is .3% less than the in-sample power, while the 5-year power is .4% more out-of-sample than in-sample. These out-of-sample ARs are 2.5 and 6.1 points higher than RiskCalc v1.0 Italy, for the 1- and 5-year models respectively.

Figure 11 shows the distribution of CRD observations by rating category in the development sample (for the Credit Cycle Adjusted EDF credit measures over the full time period). Note that 14 categories between Aa3 and Caa/C are utilized and that less than 15% of the observations are in any one category. The 1-year and 5-year distributions peak at Ba1. While not reported here, other research has shown that the distribution of the CCA EDF implied ratings changes over time with the credit cycle while the distribution of the FSO EDF implied ratings remains relatively stable over time.

FIGURE 11 EDF-implied Ratings for the 1- and 5-year models in the development sample



5 FURTHER MODEL IMPROVEMENTS

For a detailed discussion of these enhancements, refer to the Technical Document. In this section, we will briefly outline some other enhancements to the model.

5.1 Continuous Term Structure

The previous version of the RiskCalc model provided the user with two discrete default probability estimates: a 1-year and a 5-year EDF. In this version, utilizing the two point estimates for 1- and 5-year estimates we fit a Weibull function, and thus achieve a continuous term structure of EDF values for each credit. In other words, users of RiskCalc v3.1 Italy now can obtain EDF estimates for any point between 1 and 5 years. In addition, RiskCalc v3.1 provides EDF estimates for alternative definitions, such as the Forward EDF and the Annualized EDF (Table 17):

- **Cumulative EDF**

A cumulative EDF credit measure gives the probability of default over that time period. For example, a five year cumulative EDF of 13.44% means that that company has a 13.44% chance of defaulting over that five year period. The second column of Table 17 provides an example of the cumulative 1- to 5-year credit measures produced by the model.

- **Forward EDF**

The forward EDF is the probability of default between $t-1$ and t conditional upon survival until $t-1$. In other words, the 4-year Forward EDF is the probability that a firm will default between years three and four assuming the firm survived to year 3.¹⁰ The third column of Table 17 displays the forward 1- to 5-year EDF credit measures that are derived from the cumulative EDF values.

- **Annualized EDF**

The annualized EDF credit measure is the cumulative EDF value for a given period, stated on a per year basis. For Example, a company with a cumulative 5-year EDF of 13.44% would have a 5-year annualized EDF of 2.84%.¹¹ This means that the average default rate per year for a 13.44% cumulative default rate is 2.84%. The last column of Table 17 presents the annualized EDF credit measures for years 1 to 5 that are derived from the cumulative EDF values.

TABLE 17 Term Structure of EDF Credit Measures: An Example

EDF	Cumulative	Forward	Annualized
Year 1	4.23	4.23	4.23
Year 2	7.00	2.90	3.57
Year 3	9.37	2.55	3.23
Year 4	11.49	2.34	3.01
Year 5	13.44	2.20	2.84

5.2 New Analytical Tools: Relative Sensitivity

The RiskCalc v1.0 interface provides users with an analytical tool to gauge the relative impact of each variable – as a deviation from the mean of each ratio. In order to equip the users of the model with further tools, we developed relative sensitivities (also known as sensitivity multiples), which exhibit the EDF sensitivity to each of the model variables at the point of evaluation. This feature is especially useful when addressing the question of identifying variables to improve the EDF value of a company.

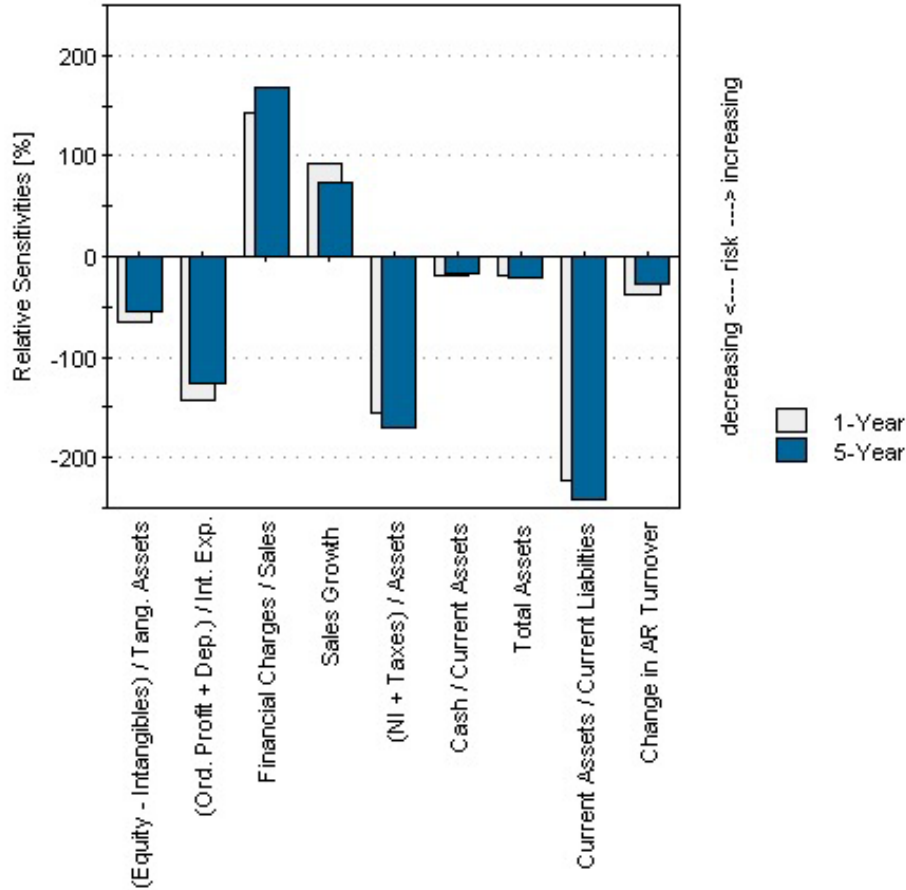
The relative sensitivity gives the impact of a small change in a variable on the EDF level of the company. It indicates which variables are most sensitive to an increase. A positive number means an increase in the variable will increase risk and a negative number indicates a decrease in risk. The percentile is the sensitivity of the variable relative to the average.

Example: A small increase in profitability ($(NI + Taxes)/ Assets$) will reduce the riskiness of the company. It is about 155% (1 year) as sensitive as the average variable (Figure 12).

¹⁰ Specifically, $FEDF_{t,t} = (CEDF_t - CEDF_{t-1}) / (1 - CEDF_{t-1})$, where $FEDF_{t,t}$ is the forward EDF from years $t-1$ to t , and $CEDF_t$ is the cumulative EDF for year t .

¹¹ Specifically, $AEDF_t = 1 - (1 - CEDF_t)^{1/t}$, where $AEDF_t$ is the annualized EDF for year t .

FIGURE 12 Relative Sensitivities



5.3 Asset Value and Volatility Calculation

One of the features of the 3.1 version of the model is that it provides an implied asset volatility. Clients of Credit Monitor and Credit Edge can use this volatility to analyze a private firm that is to go public through an IPO. Once the firm is public, the public firm model should be used, however, this model requires an asset volatility that is derived from the public share price. In the 3.1 version of the model, the asset volatility of the firm is estimated using its industry and size and a methodology that is very similar to PFM. A structural model framework is then used to solve for the implied asset value from the estimated EDF credit measure, the estimated volatility, and the firm's liability structure.

6 CONCLUSION

The RiskCalc v3.1 Italy model is based on a substantially larger database than RiskCalc v1.0 Italy and has an additional three years of data. Improved data coverage has allowed us to refine our financial statement model and achieve a very robust prediction model of private firm default behavior.

The model is more powerful than any publicly available alternatives that we have tested. We have demonstrated that the increase in power is consistent across industry sectors and size classifications as well as for different time periods. We have also shown that the power advantage is maintained both in-sample and out-of-sample.

The RiskCalc v3.1 Italy model controls for differences in the default risk across industries in the FSO mode. In addition, in the CCA mode, it adjusts the EDF level to reflect the current stage of the credit cycle in the given

industry. If default risk in a given firm's industry is high, the EDF level is adjusted upward. Likewise, when default risk is low, the EDF level is adjusted downward. This additional feature of the model increases the model power and precision and allows users to monitor their portfolios on a monthly basis.

The Moody's KMV RiskCalc v3.1 model will be very useful for financial institutions seeking to implement quantitative tools for originating loans, managing portfolio risk, and meeting regulatory requirements. It also provides them with an objective external benchmark of the risk associated with a private firm, useful in securitizing middle-market debt. Finally, as an established benchmark, RiskCalc v3.1 enables institutions to communicate with one another about their exposures.

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