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MOODY'S KMV RISKCALC™ V3.1 DENMARK

MODELING**METHODOLOGY**

ABSTRACT

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Moody's KMV RiskCalc is the Moody's KMV model for predicting private company defaults. It covers over 80% of the world's GDP, has more than 20 geographic specific models, and is used by hundreds of institutions worldwide. While using the same underlying framework, each model reflects the domestic lending, regulator, and accounting practices of its specific region.

In January 2004, Moody's KMV introduced its newest RiskCalc modeling framework, Moody's KMV RiskCalc v3.1. By incorporating both market (systematic) and company specific (idiosyncratic) risk factors, RiskCalc v3.1 is in the forefront of modeling middle-market default risk. This modeling approach substantially increases the model's predictive powers.

This document outlines the underlying research, model characteristics, data, and validation results for the RiskCalc v3.1 Denmark model.

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1 INTRODUCTION

The Moody's KMV RiskCalc v3.1 Denmark model is built using the results of extensive Moody's KMV research, including:

- Moody's KMV RiskCalc v1.0 and the Moody's KMV Private Firm Model®
- Moody's KMV Credit Research Database™ (CRD), the world's largest and cleanest private company default database
- Industry sector information, market information, and industry-specific default rates

RiskCalc v3.1 incorporates the structural and market-based comparables approach (used in the Moody's KMV Private Firm Model), and the localized financial statement-based approach (used in RiskCalc v1.0). This allows RiskCalc v3.1 to blend market-based (systematic) information with detailed firm-specific financial statement (idiosyncratic) information to enhance the model's predictive power.

RiskCalc Modes

RiskCalc v3.1 allows a user to assess the risk of a private firm in two ways: Financial Statement Only (FSO) and Credit Cycle Adjusted (CCA).

The FSO mode delivers a firm's default risk based on only financial statements and sector information, adjusted to reflect differences in credit risk across industries. In this mode, the risk assessments produced by the model are relatively stable over time.

The CCA mode adjusts the default risk by taking into account the current stage of the credit cycle. The CCA adjustment is a sector-specific factor derived directly from the Moody's KMV public firm model's distance-to-default. The CCA model reflects the market's current assessment of the credit cycle and is a forward-looking indicator of default.

The CCA adjustment is specific to the firm's sector and country and is updated monthly. The CCA mode also has the ability to stress-test Expected Default Frequency™ (EDF) credit measures under different credit cycle scenarios—a proposed requirement under Basel II.

Differences Between RiskCalc v3.1 Denmark and RiskCalc v1.0 Denmark

Since the release of RiskCalc v1.0 Denmark, Moody's KMV significantly increased the size of the database for Denmark and improved its data cleansing technologies. Due to improved data coverage, RiskCalc Denmark v3.1 includes new ratios to expand the coverage on dynamic factors of private firms' credit risk. Furthermore, the new model allows for more granular industry adjustments, credit cycle adjustments, and a complete term structures of EDF credit measures. RiskCalc Denmark 3.1 also provides new analytic tools that increase model usability and transparency. Given the advances in modeling, RiskCalc Denmark 3.1 is a more powerful predictor of default than its predecessor.

2 DATA DESCRIPTION

The source of the data for RiskCalc v3.1 Denmark is Moody's KMV CRD. Moody's KMV collects data from participating institutions, working closely with them to understand the strengths and weaknesses of the data.

2.1 Definition of Default

Moody's KMV RiskCalc provides assistance to institutions and investors for determining the risk of default, missed payments, or other credit events. The proposals for the new Basel Capital Accord (BIS II) have stimulated debates about what constitutes an appropriate definition of default. RiskCalc applies the criteria used by most of the advanced economies in the world. In model development, RiskCalc uses the local criteria for default. Accordingly, in Denmark, the events which we defined as defaults include suspension of payments, liquidation, compulsory sale, and bankruptcy. At the calibration stage, the model outputs are adjusted to ensure a consistent interpretation throughout the world. Specifically, the model outputs are converted into a term structure of actual default probabilities (1- through 5-year EDF credit measures).

2.2 Data Exclusions

Excluded Companies

The goal of the RiskCalc model is to provide an EDF credit measure for private Danish companies in the middle market. The firms and industries covered in the model must have similar default characteristics. To create the most powerful model for Danish middle-market companies, companies that did not reflect the typical company in this market were eliminated. The following types of companies are not included in the data:

- **Small companies** – For companies with Total Assets less than DKK 1,000,000 (2002 Danish Krone), future success is often linked to the finances of the key individuals. For this reason, they are not reflective of typical middle-market companies and are excluded from the database.

- **Financial institutions** – The balance sheets of financial institutions (banks, insurance companies, and investment companies) exhibit higher leverage than the typical private firm. The regulation and capital requirements of these institutions make them dissimilar to the typical middle-market company. Therefore, they are excluded from the database.

- **Real estate development companies** – The annual accounts of real estate development and investment companies provide only a partial description of the dynamics of these firms and, therefore, their likelihood to default. This is because their financial health often hinges on a particular development.[1]

- **Public sector and non-profit institutions** – Government run companies' default risks are influenced by the states' or municipalities' unwillingness to allow them to fail. As a result, their financial results are not comparable to other private firms. Not-for-profit financial ratios are different from for-profit firms, particularly with regard to variables relating to net income.

- **Start-up companies –** Our experience has shown that the financial statements for a company during its first two years are extremely volatile and are a poor reflection of the creditworthiness of the company. The special nature of start-ups is reflected in the fact that many financial institutions have separate credit departments for dealing with these companies.

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[1] The success of many types of "project finance" firms depends largely on the outcome of a particular project. We recommend using separate models for such firms. At the time of writing, this characteristic is explicitly recognized within the proposals for the new Basel Capital Accord.

Excluded Financial Statements

The financial statements of smaller companies can be less accurate and lower quality than those of larger companies. The financial statements in the CRD are cleaned to eliminate highly suspect financial statements. Plausibility checks of financial statements are conducted, such as Assets not equal to Liabilities plus Net North, and financial statements covering a period of less than twelve months. If errors are detected, those statements are excluded from the analysis.

2.3 Descriptive Statistics of the Data

Overview of the Data

The extensive data on both non-defaulting and defaulting companies contained in Moody's CRD increased substantially since RiskCalc v1.0 was developed. In addition to the increase in time-series data, there has been an increase in the number of participants in the CRD.

Figure 1 presents the distribution of Danish financial statements and defaults by year in the CRD. Table 1 summarizes the data used in the development, validation, and calibration of the RiskCalc v3.1 Denmark model.

FIGURE 1 Date Distribution of Danish Financial Statements and Defaults

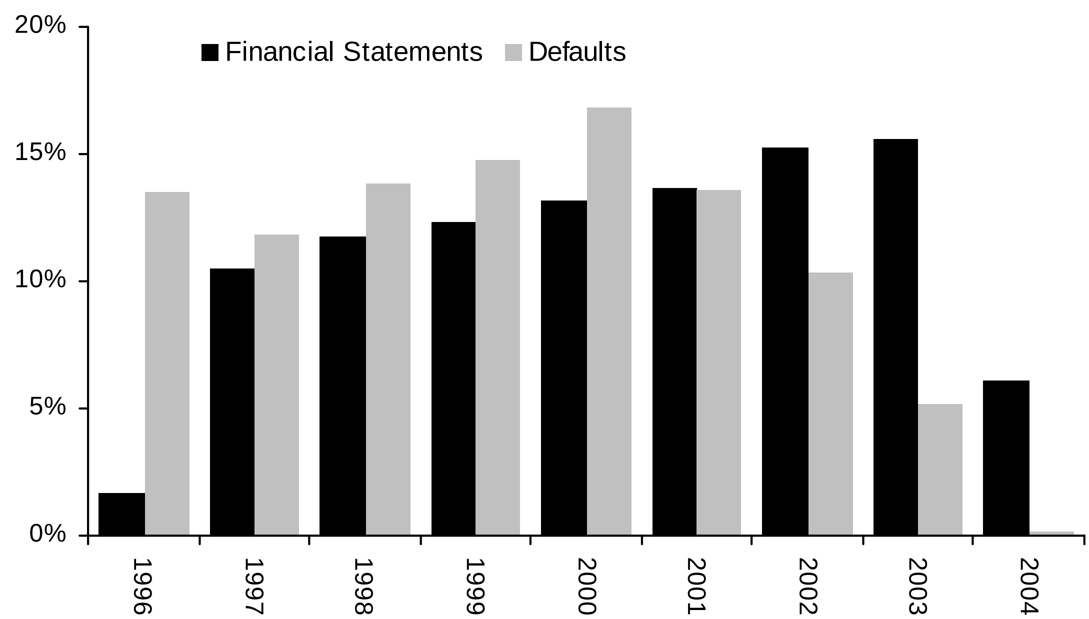


TABLE 1 Information on Danish Private Firm Sample Data

Danish Private Firms	RiskCalc v1.0 Denmark	RiskCalc v3.1 Denmark	Change
Financial statements	118,639	155,938	↑ 31%
Unique number of firms	29,997	48,008	↑ 60%
Defaults	1,542	3,259	↑ 111%
Time period	1996–2001	1996–2004	+3 year

Robustness of the Data

In building a model, potential database weaknesses need to be examined. Not only does the database need to cover a large number of firms and defaults, but the defaults also need to be distributed among industries and company types covered. For example, if the database has significant numbers of small firms or firms in one particular industry and there are not sufficient defaults in those areas, the model may not be a good default predictor. The CRD used in developing the RiskCalc models has addressed both of these issues.

Figure 2 presents the distributions of Danish firms by industry and the proportion of defaults in each industry. Figure 3 presents the distributions by the size of firms measured as Total Assets and in 2002 Danish Krone. These figures demonstrate that the proportion of defaults in any one industry group or size group is comparable to the proportion of firms in these groupings. The size distribution shows that about 55% of the firms hold assets less than 4 million Danish Krones.

FIGURE 2 Distribution of Danish Defaults and Firms by Industry

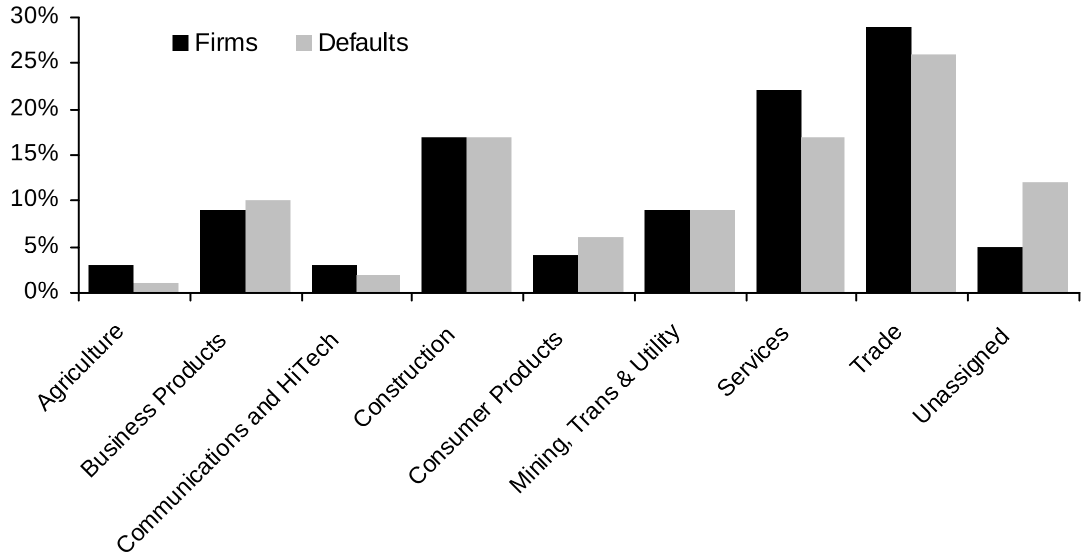
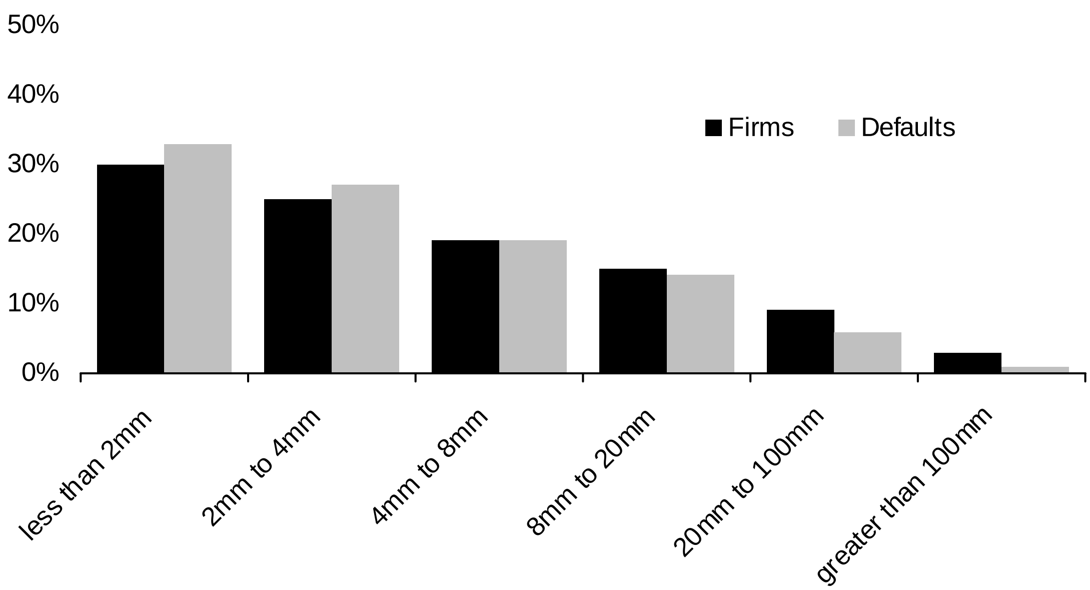


FIGURE 3 Size (as Total Assets in Danish Krone) Distribution of Defaults and Firms



2.4 Cleaning the Data

In the development of a RiskCalc model, the first step is the collection of a large and appropriate database. In addition, data needs to be “cleaned” so that it is representative of the actual risk of the firms covered. Moody’s KMV developed techniques for cleaning the database to improve the model results.

2.5 Central Default Tendency

Because most companies do not default, companies that do default are relatively rare and thus more valuable in building a default prediction model. Much of the lack in default data is because of the data storage issues within financial institutions such as, defaulting companies being purged from the system after troubles begin, not all defaults being captured, or other sample errors. Also, if the date of default is uncertain, the financial statement associated with the firm may be excluded from model development, depending on the severity of the problem. This can result in a sample that has lower default rates than occur in the general population. If the underlying sample is not representative, then it needs to be adjusted for the true central default tendency. When default definitions used in the data sample understate the defaulting population, as is the case with Denmark, the central default tendency can be used to realign the default rates.

The estimate of long-run aggregate probabilities of default (or central default tendency) is important as an anchor for a model. The best estimation of default probability is a ratio that would reflect the number of obligors that defaulted in one year compared with the total obligors at the beginning of that year. Often these types of data are not available.

The estimate of the central default tendency for Denmark is based on several sources.

- We examined loan loss provision data from the Organization for Economic Co-operation and Development (OECD) and provisioning data from financial statements of large Danish banks.
- We examined bankruptcy data from Statistics Denmark.
- We ensured that the central default tendency exceeded the default rates observed in our development sample.

The multiple sources of external data lead us to an estimate of 1.9% as the central tendency figure for the 1-year model. This estimate is consistent with the average probability of default from the RiskCalc v1.0 Denmark model on the development sample.

Calculating a 5-year Central Default Tendency

There is a lack of publicly available data for direct calculation of the central tendency rate of a cumulative 5-year default probability. Based on extensive Moody's KMV research, a 5-year cumulative default tendency is derived from the 1-year estimate. This research, combined with the information provided by the CRD, shows that the 5-year cumulative default rate is, on average, four times the level of the 1-year default rate. Therefore, 7.6% is used as the central default tendency for the 5-year model.

Central Default Tendency in FSO and CCA Modes

In the FSO model, the central default tendency remains constant over time. In CCA mode, the central default tendency is equal to the central default tendency of the FSO mode when the effects of the credit cycle are neutral. When the forward-looking prediction of the credit cycle indicates increasing default risk, the central default tendency of the CCA mode will be larger, and when the effects of the credit cycle indicate reducing default risk, the central default tendency will be smaller.

3 MODEL COMPONENTS

The RiskCalc v3.1 model incorporates various components to determine the EDF credit measure. The inputs to the model include a selection of the financial ratios and transforms of those ratios, the inclusion of industry information, and the credit cycle adjustment.

The development of a RiskCalc model involves the following steps:

1. Choosing a limited number of financial statement variables for the model from a list of possible variables.[2]
2. Transforming the variables into interim probabilities of default using non-parametric techniques.
3. Estimating the weightings of the financial statement variables, using a probit model, combined with industry variables.
4. Creating a (non-parametric) final transform that converts the probit model score into an actual EDF credit measure.

In FSO mode, the models are based on the following functional form:

$$FSO\ EDF = F\left(\Phi\left(\sum_{i=1}^{N} \beta_i T_i(x_i) + \sum_{j=1}^{K} \gamma_j I_j \right) \right)$$

where $x_1,...,x_N$ are the input ratios; $I_1,...,I_K$ are indicator variables for each of the industry classifications; β and γ are estimated coefficients; Φ is the cumulative normal distribution; F and $T_1,...,T_N$ are non-parametric transforms; and FSO EDF is the financial-statement-only EDF credit measure.[3] The Ts are the transforms of each financial statement variable, which capture the non-linear impacts of financial ratios on the default likelihood. (This is shown in Figure 4 and discussed in detail later in the document.) F is the final transform (i.e. the final mapping). The final transform captures the empirical relationship between the probit model score and actual default probabilities. We describe the final transform as calibrating the model score to an actual EDF credit measure. The difference between the FSO EDF and the credit cycle adjusted EDF is that in CCA mode the final transform is adjusted to reflect our assessment of the current stage of the credit cycle, while in FSO mode it remains constant.

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[2] These variables are often ratios, but not always. For example, one measure of profitability is Net Income to Total Assets, which is a ratio, and one measure of size is Inflation Adjusted Total Assets, which is not a ratio.

[3] By non-parametric, we mean that the $T(x_i)$ is a continuous function of x not requiring a specification of a specific closed (or parametric) functional form. We estimate these transforms using a variety of local regression and density estimation techniques.

3.1 Financial Statement Variables

Selecting the Variables

Our variable selection process starts with a long list of possible financial statement variables. The working list of ratios is divided into groups that represent different underlying concepts regarding a firm's financial status (see Table 2). A model is then built with at least one variable per group. When it is possible to increase model performance while maintaining model robustness, several variables from each group will be used in the model. We ask the following questions when deciding which variables to include in the final model:

- Is the variable readily available?
- Are the definitions of the inputs to the variable unambiguous?
- Is the meaning of the variable intuitive?
- Does the variable predict default activity?
- Is the variable not highly correlated with other variables in the model?

TABLE 2 Groupings of Financial Statement Ratios

Examples of ratios in the **profitability** group include net income, net income less extraordinary items, profit before tax, and operating profit in the numerator; and total assets, tangible assets, fixed assets and sales in the denominator. → *High profitability reduces the probability of default.*

Examples of ratios in the **leverage (or gearing)** group include liabilities to assets and long-term debt to assets. → *High leverage increases the probability of default.*

Debt coverage is the ratio of cash flow to interest payments or some other measure of liabilities. → *High debt coverage reduces the probability of default.*

Growth variables are typically the change in ROA and sales growth. These variables measure the stability of a firm's performance. → *Growth variables behave like a double-edged sword: both rapid growth and rapid decline (negative growth) tend to increase a firm's default probability.*

Liquidity variables include pure cash or cash and marketable securities to assets, the current ratio, and the quick ratio. These variables measure the extent to which the firm has liquid assets relative to the size of its liabilities. → *High liquidity reduces the probability of default.*

Activity ratios include inventories to sales and accounts receivable to sales. These ratios may measure the extent to which a firm has a substantial portion of assets in accounts that may be of subjective value. For example, a firm with large inventories may not be selling its products and may have to write off these inventories. → *A large stock of inventories relative to sales increases the probability of default; other activity ratios have different relationships to default.*

Size variables include sales and total assets. These variables are normally deflated to a specific base year to ensure comparability (e.g., total assets are measured in 2002 Euros). → *Large firms default less often.*

TABLE 3 Financial Statement Variables used in RiskCalc v3.1 Denmark

Category	Definition
Activity	**Trade Creditors to Turnover:** Trade Creditors / Net Turnover
Debt Coverage	**Debt Service Coverage**: Operating Cash Flow / Financial Expenses
Growth	**Sales Growth**: Net Turnover(t)/Net Turnover(t-1) – 1 **Change in ROA**: ROA(t) – ROA(t-1)
Leverage/Gearing	**Liabilities To Assets**: Total Liabilities / Total Assets
Liquidity	**Current Ratio:** Total Current Assets / Short-term Debts) **Current Asset Structure**: Cash at Bank and in Hand / Total Current Assets
Profitability	**EBT to Assets**: Pre-Tax P&L / Total Assets
Size	**Size**: Total Real Assets in 2002 Danish Krone

Variable Transforms

After the variables are selected, they are transformed into a preliminary EDF value. Figure 4 presents the transformations used in the model. The horizontal axis gives the percentile score of the ratio and the vertical axis gives the default probability of that ratio in isolation (univariate). The percentile score represents the percent of the database that had a ratio below that of the company (e.g., if ROA is in the 90th percentile that means that 90% of the sample has an ROA lower than that firm).

The shape of the transformation indicates how significantly a change in level impacts the EDF value. If the slope of the transform is steep, a small change will have a larger impact on risk than if the slope were flat.

- For the **Profitability** group, the transforms for EBT to Assets is downward sloping. For this ratio the slope of the transform approaches zero as profitability becomes large (Figure 4). Such a transform indicates that more profitable firms have lower default probabilities, but the impact diminishes as profitability increases.

- For the **Leverage/Gearing** group, the transform is upward sloping (Figure 4). The transform for Total Liabilities to Total Assets is upward sloping because increasing Total Liabilities relative to Total Assets increases default risk. The slope becomes steeper as leverage increases, indicating that highly levered firms' risk is more sensitive to changes in leverage.

- For the **Liquidity** group, the transforms for the Current Ratio and the Current Asset Structure ratio are downward sloping. The Current Ratio reflects the extent to which a firm can cover its short-term debts with liquid funds (Current Assets). Current Asset Structure (Cash at Bank and in Hand / Total Current Assets) measures the portion of Current Assets that are immediately available for use. For Current Asset Structure, the slope of the transforms is similar across the percentile space; therefore, changes in either direction from the median imply an equal change in risk (Figure 4). For the Current Ratio, the slope of the transform is flat and then becomes downward sloping, which indicates that if the current ratio is above the 25% percentile, the firm's credit risk decreases as the current ratio increases.

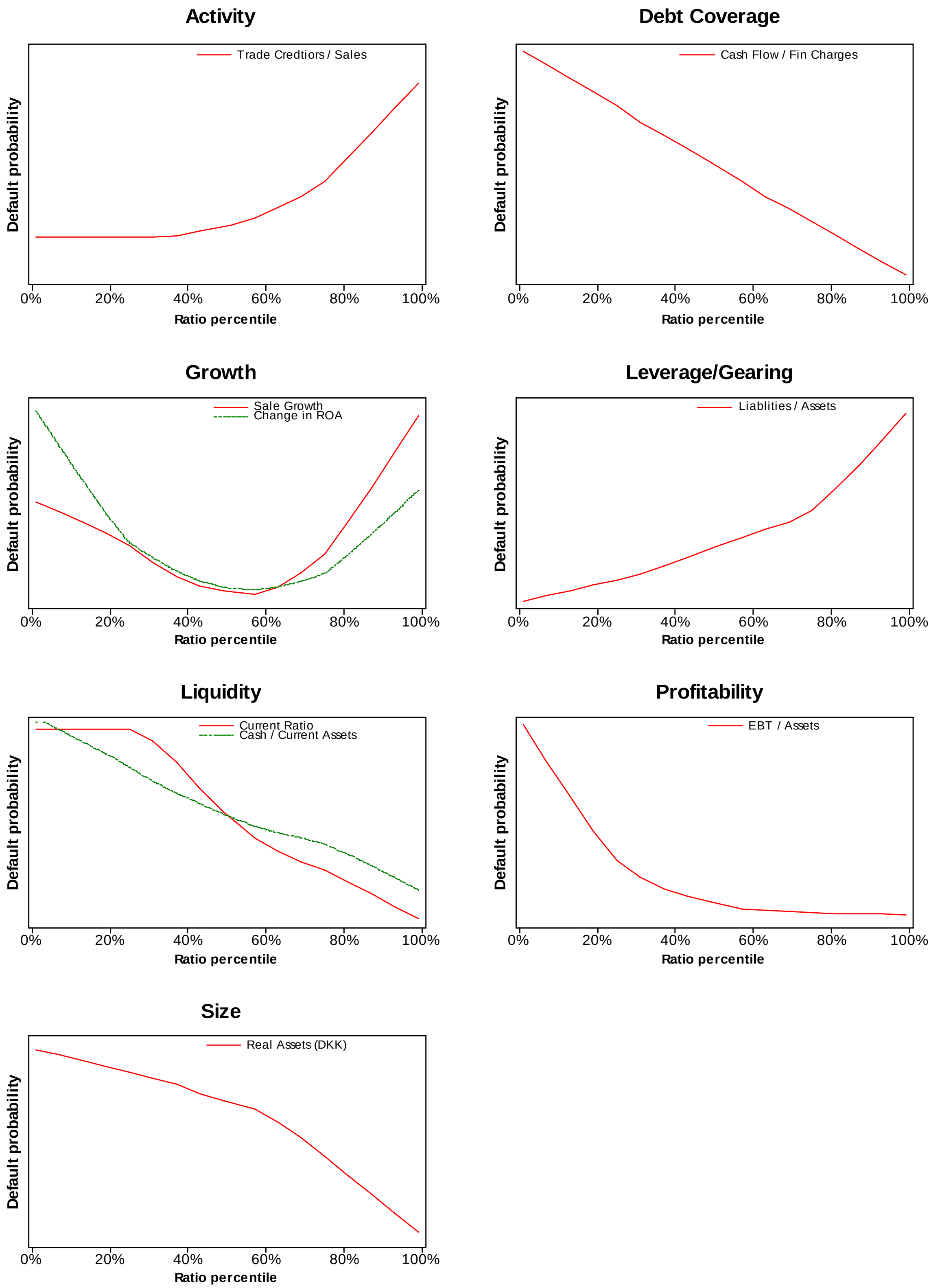
- For the **Activity** group, the transform for the Trade Creditors to Net Turnover (Net Sales) ratio is upward sloping. The slope of the transform is flat and then becomes steeper (Figure 4). This shape indicates that risk levels are fairly insensitive to movements in the ratio below the 40% percentile.

- The **Size** variable is inflation adjusted Total Assets (2002 Danish Krone). This variable's transformation is downward sloping (Figure 4). This indicates that larger firms have lower default probabilities.

- The **Debt Coverage** variable is Operating Cash Flow over Financial Expenses. This transform is downward sloping indicating that large values of cash flow relative to Financial Expenses lower the probability of default. The slope decreases as debt coverage decreases (Figure 4).

- The **Growth** variables are Sales Growth and Change in ROA. Both of them are "U-shaped," indicating that large increases or decreases in Sales and ROA are associated with higher default probabilities, while stable Sales and ROA year-upon-year decreases the probability of default (Figure 4).

FIGURE 4 Transformations of Financial Statement Variables Used in the Model



3.2 Model Weights

Importance

The relative value of each variable used in calculating an EDF credit measure is important in understanding a company's risk. The non-linear nature of the model makes the weight of the variables more difficult to determine since the actual impact on the risk depends on the coefficient, the transformation shape, and the percentile ranking of the company. The model weights, therefore, are calculated based on the average EDF value for the transformation and its standard deviation. A variable with a flat transformation could have a low weight, even if the coefficient is large (Figure 4).

Calculation of Weights

To calculate the weighting of a variable, the EDF credit measure for a theoretical firm with all its variables at the average transformation values is computed. The variables are then increased one at a time by one standard deviation. The EDF change for each variable (in absolute value) is computed and added together. The relative weight of each variable is then calculated as the EDF level change for that variable as a percent of the sum of EDF level changes across all variables. This gives the variable with the biggest impact on the EDF level the biggest weight, and the variable that has the smallest impact on the EDF level the smallest weight. Since the weights are a percentage of the total change in EDF levels, they sum 100%. The weight of each category is the sum of the weights of each variable in that category. Table 4 presents the weights in RiskCalc v3.1 Denmark. The most important categories are Leverage/Gearing, Activity and Liquidity.

TABLE 4 Risk Drivers in RiskCalc v3.1 Denmark

Category	Weights
Leverage/Gearing	27%
Activity	19%
Liquidity	18%
Growth	12%
Profitability	11%
Size	9%
Debt Coverage	4%

3.3 Industry Adjustments

While the variables included in the RiskCalc model explain most of the risk factors, the relative importance of the variables can be different among industries. In the FSO mode of RiskCalc v3.1 Denmark, the EDF value is adjusted for industry effects. Table 5 presents the increase in model power and accuracy from introducing industry controls into the FSO model. Both the power and the accuracy of the EDF credit measure increase, as measured by the accuracy ratio and the gain in log likelihood. The gain in likelihood indicates that the industry controls are especially important in producing an accurate EDF. Table 6 presents the average EDF value by industry for the development sample.

TABLE 5 Increase in Model Power and Accuracy from Introducing Industry Controls

	1-year Model		5-year Model	
	Accuracy Ratio	Relative increase in Log Likelihood	Accuracy Ratio	Relative increase in Log Likelihood
FSO mode without industry controls	72.2%		59.0%	
FSO mode with industry controls	72.6%	74***	59.7%	126***

*** Indicates a P-value of less than 0.01 percent.

In this table, and hereafter, Accuracy Ratio (or AR) is the measure of the model's ability to rank order credits. Increases in log likelihood measure the extent to which the model's EDF values match observed default rates. For further details, see Dwyer and Stein (2004), *Technical Document on RiskCalc v3.1 Methodology* (Technical Document).

TABLE 6 Average EDF Credit Measure by Sector

Sector	Average 1-year EDF	Average 5-year EDF
Agriculture	1.55%	6.78%
Business Products	2.67%	9.91%
Telecommunications and Hi Tech	3.08%	7.88%
Construction	2.45%	9.82%
Consumer Products	2.31%	9.20%
Mining, Transportation, Utilities and Natural Resources	2.00%	8.14%
Services	1.89%	6.40%
Trade	1.95%	7.49%

3.4 Credit Cycle Adjustment

EDF credit measures are impacted not only by the financials of a company, but also by the general credit cycle in the economy. To capture this effect, RiskCalc v3.1 Denmark includes a credit cycle adjustment factor. The credit cycle adjustment is designed to incorporate the current credit cycle into the estimate of private firm default risk.

Selecting an Adjustment Factor

The RiskCalc v3.1 model uses the distance-to-default calculation from the Moody's KMV public firm model. This measure is specifically designed to be a forward-looking indicator of default risk. It extracts signals of default risk from the stock market performance of individual firms (cf., Bohn and Crosbie, 2003). This measure was chosen because it is available for a large universe of industries and it has been extensively validated.

If the distance-to-default for public firms in an industry indicates a level of risk above the historical average for that industry, then the private firms' EDF values in that industry are adjusted upward. Conversely, if the level of risk is below the historical average, then the private firms' EDF values are adjusted downward. When the credit cycle adjustment factor is neutral, the CCA EDF value coincides with the FSO EDF value.

Adjustment Factor Used in the Model

For the Danish model, the distance-to-default factor for each industry is a weighted average of two indices. The average is based on the aggregation of distance-to-default in each industry for all public firms in the Nordic region countries and public firms in a basket of fifteen European countries[4]. The weight on the Nordic factor is industry specific and determined by the market value of assets of Nordic firms in each industry relative to all firms in the basket. In the event that a firm cannot be associated with a specific industry, the model uses a credit cycle adjustment based on an aggregation of all public firms in the Nordic region and the associated countries.

The distance-to-default factor is meant to be a *forward-looking* indicator of default risk. One way to measure the markets current assessment of credit risk is to examine credit spreads on corporate bonds. When the market expects higher levels of default on public debt, the yield spread over a risk-free bond will increase to compensate for the extra risk. Figure 5 presents the evidence of the Nordic distance-to-default factor and yield spreads on Western European Corporate Bonds. The distance-to-default factor is inverted when graphed so that higher values on the graph indicate higher probabilities of default for Nordic public firms. We would expect a concurrent relationship between the series since both are forward-looking, which is what the figure shows.

Figure 6 provides evidence of the relationship between the distance-to-default factor and public default rates in Europe as measured by Moody's KMV[5]. Similar to credit spread evidence, the factor is a *forward-looking* measure of the probability of default for public European firms. Overall, the evidence shows that the distance-to-default factor is a strong predictor of economic conditions in each industry and will adjust the probabilities of default to reflect the position in the credit cycle.

[4]In this context, a public company is a company with publicly traded equity. The European index includes Austria, Belgium, Denmark, Finland, France, Germany, Greece, Italy, Luxemburg, Netherlands, Norway, Portugal, Spain, Sweden, and Switzerland.

[5] The public default rate is based on the same fifteen countries included in the distance-to-default factor calculation.

FIGURE 5 Nordic DD Factor and Western Europe Corporate Yield Spreads: Jan. 2000–Feb. 2006

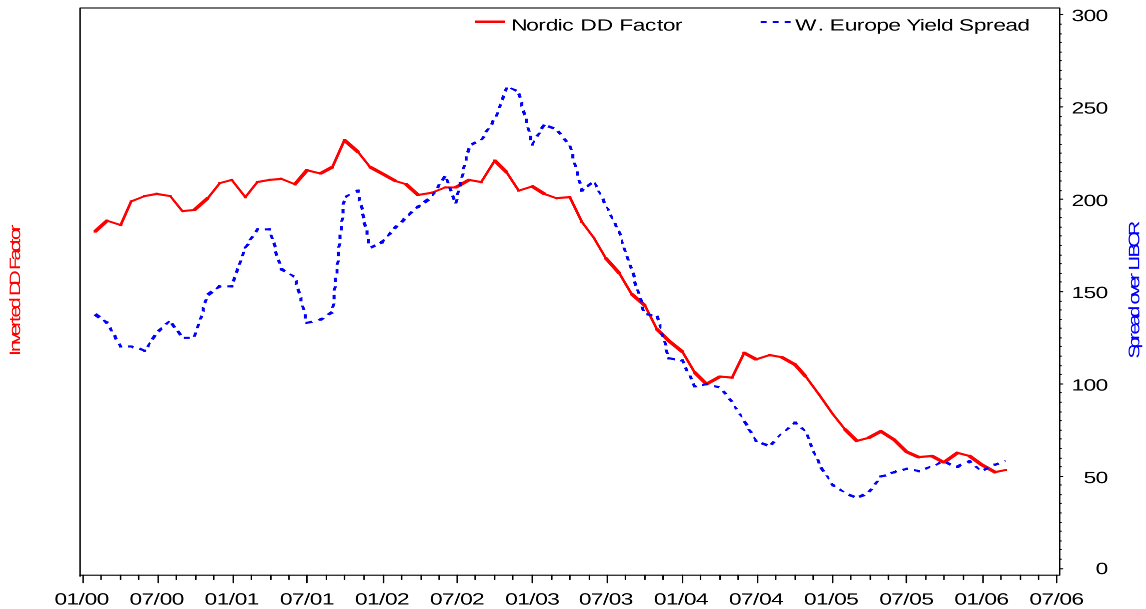


Figure 5 displays the DD factor (red solid line) against the historical credit spread levels in (blue dotted line). Bond prices and yields are from Reuters EJV and the yield spread is over the benchmark LIBOR rate. The spread statistics are complied using Moody's KMV CreditEdge for the Western Europe Corporate Bond Group.

FIGURE 6 Nordic DD Factor and Europe Public Default Rates: 1997–2005

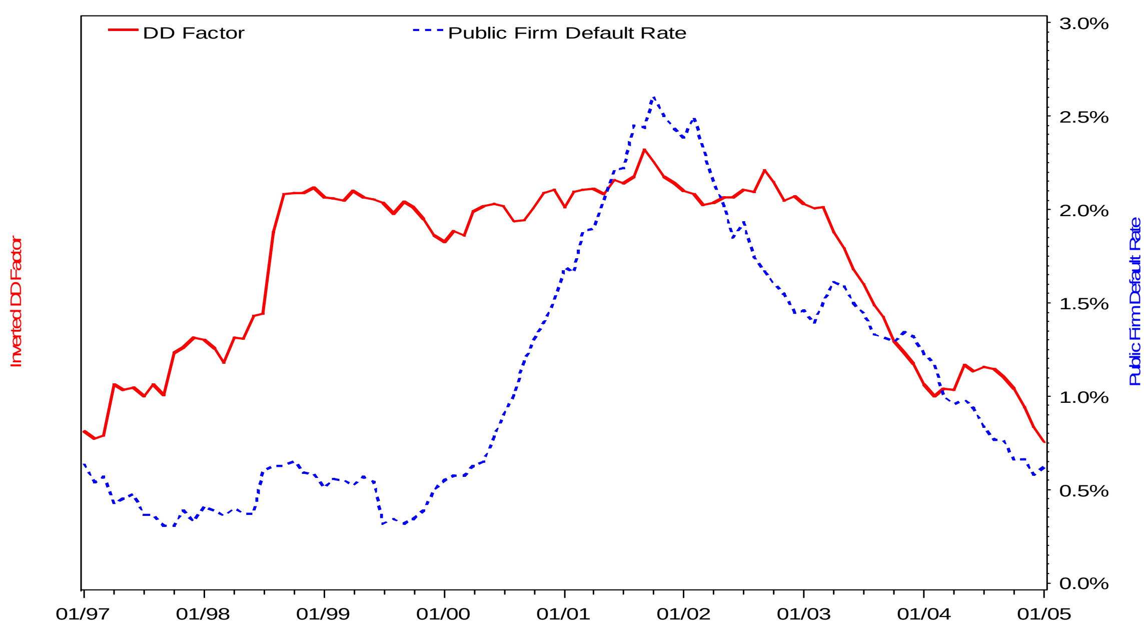


Figure 6 displays the Nordic DD factor (red solid line) against the historical public firm default rate for Europe (blue dotted line). The DD factor increases in anticipation of the increase in default activity.

4 VALIDATION RESULTS

After a model is developed, it must be shown to be effective in predicting defaults. In this section, we present testing results on the model's ranking power (the model's ability to sort credits from worst to best).

The tests need to check not only the model effectiveness, but also its robustness and how well it works on data outside the sample. To do out-of-sample testing, we performed walk-forward and *k*-fold analyses. The results of the testing show that the model is uniformly more powerful than other models across different time periods, sectors, and size classifications.

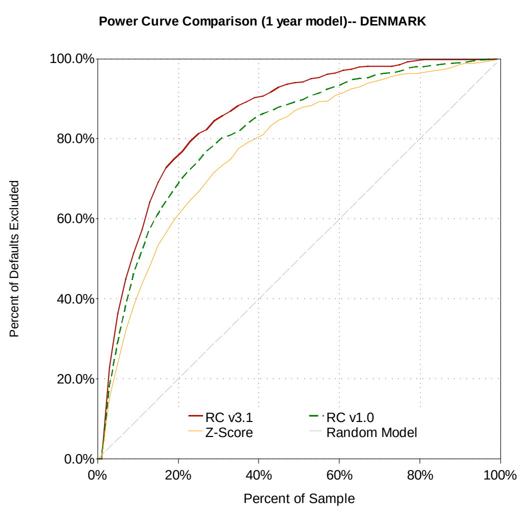
4.1 Increase in Overall Model Power and Accuracy

Table 7 presents the in-sample overall measures of power for RiskCalc v3.1 Denmark versus alternative models. With the credit cycle adjustment, the model's performance improves by more than six percentage points of accuracy ratio at both the 1-year horizon and the 5-year horizon compared with RiskCalc v1.0 Denmark. Relative to other available alternatives, the results were more dramatic. The new RiskCalc v3.1 model outperformed the Z-score model (Altman, Hartzell and Peck, 1995) by more than thirteen percentage points at both the 1-year horizon and the 5-year horizon[6].

TABLE 7 Power Enhancements of the new RiskCalc v3.1 Denmark Model

	1-year Model	5-year Model
	Accuracy Ratio	Accuracy Ratio
RiskCalc v3.1	71.7%	57.8%
RiskCalc v1.0	65.4%	51.1%
Z-score	58.3%	44.4%

FIGURE 7 Power of Alternative Models (1- and 5-year) — Denmark



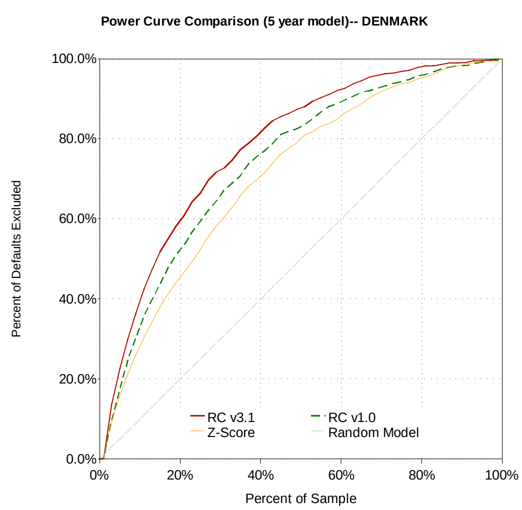


Figure 7 presents the cumulative accuracy profiles for the 1- and 5-year models corresponding to Table 7. The power improvements are uniformly significant across different regions of the distribution relative to RiskCalc v1.0.

[6] The corresponding accuracy ratios are 72.6% (FSO) vs. 65.4% (RiskCalc v1.0) for the 1-year horizon and 59.7% (FSO) vs. 51.1% (RiskCalc v1.0) for the 5-year horizon.

4.2 Correlations and Variance Inflation Factors

To ensure model robustness, the model must be tested for excessive multicollinearity, which occurs if a number of the variables used in the model are highly correlated. Excessive multicollinearity can cause instability in parameter estimates. In order to check for this issue, the correlation coefficients (Table 8) for the financial statement ratios in the model and the variance inflation factors (Table 9) are computed on the transformed variables (see Figure 4).[7]

Model Results

TABLE 8 Correlations Among the Transformed Input Factors (Spearman Rank)

	Trade Creditors to Turnover	Size	Debt Service Coverage	Liabilities to Assets	Sales Growth	Change in ROA	Current Ratio	Current Asset Structure	EBT to Assets
Trade Creditors to Turnover	1.0								
Size	0.01	1.0							
Debt Service Coverage	0.15	0.00	1.0						
Liabilities to Assets	0.19	0.02	0.25	1.0					
Sales Growth	0.09	0.08	0.09	0.04	1.0				
Change in ROA	0.04	0.16	0.12	0.08	0.18	1.0			
Current Ratio	0.13	0.05	0.17	0.59	0.01	0.04	1.0		
Current Asset Structure	0.08	-0.14	0.20	0.24	-0.01	-0.01	0.19	1.0	
EBT to Assets	0.15	0.07	0.51	0.36	0.06	0.32	0.24	0.15	1.0

The Variance Inflation Factors (Table 9) for the financial statement variables represent how much of the variation in one independent variable can be explained by all the other independent variables in the model. The correlation coefficient, however, measures only the relationships between two variables. The VIF levels are

[7] For further definitions and technical discussion of the testing procedures in Section 4, refer to the Technical Document.

relatively low, indicating that the collinearity between the variables is relatively low.[8] The two ratios with the highest VIFs have the second highest correlation in Table 8. The highest correlation among the variables is between the Current Ratio and Liabilities to Assets.

TABLE 9 Variance Inflation Factors

Variable	VIF
Liabilities to Assets	1.74
EBT to Assets	1.64
Current Ratio	1.58
Debt Service Coverage	1.43
Change in ROA	1.19
Current Asset Structure	1.15
Size	1.09
Sales Growth	1.07
Trade Creditors to Turnover	1.07

4.3 Model Power by Industry and Size Groups

It is important to test the power of a model not only overall, but also among different industry segments and firm sizes. Table 10 and Table 11 present the power comparisons by sector for the 1-year and 5-year models, respectively.

RiskCalc v3.1 Denmark outperforms both RiskCalc v1.0 Denmark and Z-score in all sectors at the 1-year horizon. The highest power in the 1-year model is found in Trade (78.4%) while the lowest is found in Unassigned (61.2%). At the 5-year horizon (Table 11) the highest power is in Trade (64.2%) and the lowest is in Unassigned (51.8%).

[8] As Woolridge (2000) shows VIF is inversely related to the tolerance value ($1-R^2$), such that a VIF of 10 corresponds to a tolerance value of 0.10. Clearly, any threshold is somewhat arbitrary and depends on the sample size. Nevertheless, if any of the R^2 values are greater than 0.75 (so that VIF is greater than 4.0), we would typically suspect that multicollinearity could be a problem. If any of the R^2 values are greater than 0.90 (so that VIF is greater than 10) we then conclude that multicollinearity is likely to be a serious problem.

TABLE 10 Model Power by Industry 1-year Model

	Percentage of Defaults	AR* v3.1	AR v1.0	AR Z-score
Business Products	7.7%	72.8%	69.2%	65.3%
Telecommunications and Hi Tech	2.6%	72.1%	68.6%	48.0%
Construction	17.9%	72.2%	66.1%	55.1%
Consumer Products	4.0%	68.0%	60.5%	45.3%
Mining, Transportation, Utilities and Natural Resources	8.7%	77.5%	74.5%	61.7%
Services	24.9%	70.6%	63.5%	58.1%
Trade	16.6%	78.4%	68.4%	67.7%
Unassigned	17.6%	61.2%	56.9%	45.8%

*AR = accuracy ratio

TABLE 11 Model Power by Industry 5-year Model

	Percentage of Defaults	AR* v3.1	AR v1.0	AR Z-score
Business Products	7.5%	58.4%	47.7%	47.0%
Telecommunications and Hi Tech	2.0%	57.0%	49.3%	28.1%
Construction	17.9%	56.1%	49.0%	41.2%
Consumer Products	3.8%	61.1%	57.5%	56.4%
Mining, Transportation, Utilities and Natural Resources	7.7%	55.0%	56.5%	49.3%
Services	22.7%	56.9%	51.7%	45.5%
Trade	17.1%	64.2%	55.4%	54.5%
Unassigned	21.3%	51.8%	47.5%	33.1%

*AR = accuracy ratio

Table 12 and 13 present the power comparisons by firm size (Total Assets in 2002 Denmark Krone) for the 1-year and 5-year models, respectively. RiskCalc v3.1 Denmark outperforms both RiskCalc v1.0 Denmark and Z-score in all size groups. The highest power in the 1-year model is found in the over 50MM group of firms. The highest power in the 5-year model is found in the 2.5MM to 10MM group, and the lowest is in the over 50MM group.

TABLE 12 Model Power by Size (Total Assets in 2002 Denmark Krone) 1-year model

	Percentage of Defaults	AR* v3.1	AR v1.0	AR Z-score
1MM to 2.5MM	45.2%	68.2%	64.8%	53.6%
2.5MM to 10MM	35.6%	70.6%	63.6%	55.5%
10 MM to 50MM	13.9%	68.3%	58.8%	56.3%
Over 50 MM	5.3%	73.0%	71.8%	62.3%

*AR = accuracy ratio

TABLE 13 Model Power by Size (Total Assets in 2002 Denmark Krone) 5-year model

	Percentage of Defaults	AR* v3.1	AR v1.0	AR Z-score
1MM to 2.5MM	47.6%	56.9%	53.4%	42.9%
2.5MM to 10MM	36.7%	59.7%	52.8%	44.9%
10 MM to 50MM	11.1%	50.0%	40.8%	39.9%
Over 50 MM	4.5%	47.6%	44.9%	33.3%

*AR = accuracy ratio

4.4 Power Performance Over Time

Since models are implemented at various points in a business cycle by design, power tests by year (Table 14 and Table 15) were conducted to examine whether the model performance is excessively time dependent.

Table 14 and Table 15 present the results from this analysis at the 1- and 5-year horizons, respectively. The AR of RiskCalc v3.1 Denmark is compared with RiskCalc v1.0 Denmark and Z-score for each year. As shown in these tables, RiskCalc v3.1 consistently outperforms both by a considerable margin except for 1996 (1-year model).

TABLE 14 Model Power over Time: 1-year Horizon

	Percentage of Defaults	AR* v3.1	AR v1.0	AR Z-score
1996	10.1%	46.2%	46.4%	40.7%
1997	11.3%	77.7%	67.4%	58.9%
1998	14.8%	78.5%	68.1%	66.0%
1999	14.2%	62.6%	55.6%	47.3%
2000	16.6%	70.3%	61.0%	53.5%
2001	15.5%	68.1%	61.8%	50.9%
2002	11.8%	63.6%	59.0%	48.5%
2003	5.8%	72.0%	65.1%	59.4%

*AR = accuracy ratio

TABLE 15 Model Power over Time: 5-year Horizon

	Percentage of Defaults	AR* v3.1	AR v1.0	AR Z-score
1996	18.4%	48.5%	43.4%	36.4%
1997	16.8%	64.8%	56.7%	45.6%
1998	18.1%	64.2%	56.7%	50.4%
1999	16.6%	57.7%	48.5%	40.6%
2000	14.3%	63.5%	52.7%	44.2%
2001	10.0%	62.2%	55.5%	44.0%
2002	5.9%	64.4%	58.8%	49.3%

*AR = accuracy ratio

4.5 Out of Sample Testing: *k*-fold Tests

The model exhibits a high degree of power in distinguishing good credits from bad ones (in Table 7), but whether this power is attributable to the overall model effectiveness or the impact of a particular sub-sample also needs to be tested. A standard test for evaluating this is the "*k*-fold test," which divides the defaulting and non-defaulting companies into *k* equally-sized segments. This yields *k* equally-sized observation sub-samples that exhibit the identical overall default rate and are temporally and cross-sectionally independent. The model is then run on *k-1* sub-samples and these parameter estimates are used to score the *k-th* sub-sample. We repeat this procedure for all possible combinations, and put the *k* scored "out-of-sample" sub-samples together to calculate an accuracy ratio on this combined data set.

Table 16 summarizes the *k*-fold test results (with *k*=5). The reported figures are the accuracy ratios by the corresponding sample and time horizons. The out-of-sample model consistently out-performs RiskCalc v1.0 Denmark. Figure 8 presents the cumulative accuracy profiles associated with the overall "out-of-sample" results against the in-sample results. The model performance is maintained both in- and out-of-sample in the *k*-fold analysis.

Results

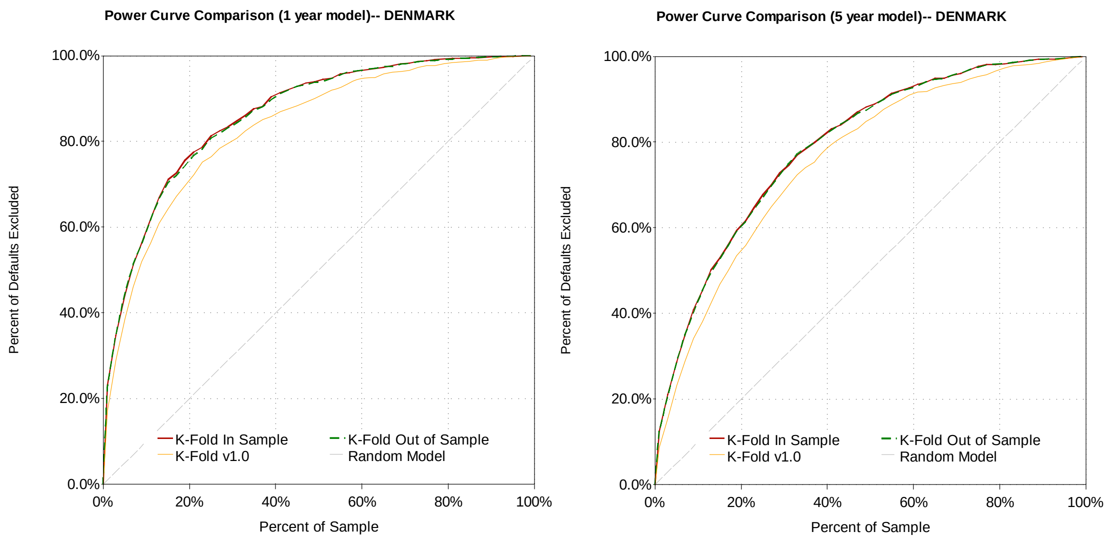
The in- and out-of-sample plots are virtually indistinguishable at both the 1- and 5-year horizons in Figure 8. The difference in AR between the overall in-sample and out-of-sample results is not larger than 50 basis points in both cases. Furthermore, RiskCalc v3.1 Denmark outperforms RiskCalc v1.0 Denmark in an out-of-sample context at both the 1- and 5-year horizons (Table 16).

TABLE 16 RiskCalc v3.1 Denmark k-fold Test Results

	Out-of-Sample AR*		RiskCalc v1.0	
	1-year AR	5-year AR	1-year AR	5-year AR
Subsample 1	69.4%	61.8%	64.3%	55.0%
Subsample 2	69.9%	59.3%	64.7%	52.1%
Subsample 3	64.5%	58.0%	61.0%	54.3%
Subsample 4	67.5%	59.8%	58.8%	50.3%
Subsample 5	69.8%	60.7%	60.5%	51.9%
K-fold Overall	71.2%	57.4%		
In-sample AR	71.7%	57.8%	65.4%	51.1%

*AR = accuracy ratio

FIGURE 8 RiskCalc v3.1 Denmark *k*-fold



The *k*-fold testing does not control for time dependence. Each of the *k* sub-samples contains data from all periods. As a result, if there were a particularly high period of default rates, this would be included in each of the *k* samples. Such testing does not give a true sense of the how the model would have performed during those volatile periods because the model is estimated with full information on those time periods.

4.6 Walk-Forward Tests

An alternative out-of-sample test developed by Moody's KMV is a *walk-forward* analysis, which is designed along similar lines as the *k*-fold test, except that it controls for the effects of time. The model is estimated up to a certain year and the parameter estimates are then used to score the observations in the *next* year. These model scores are *out-of-time*. The model is re-estimated including one more year of data. We then repeat the analysis for the next year, and continue until the end of the sample. These out-of-sample out-of-time scores are combined into a single prediction set so that the accuracy ratio and the power curve can be calculated for the combined set. The out-of-sample accuracy ratio is then compared to the corresponding in-sample accuracy ratio and power curve.

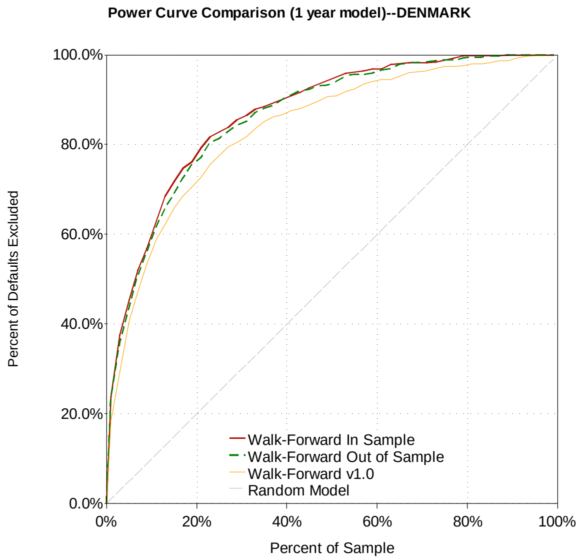
No data from a future period is used in fitting the model, and only data from future periods is used for testing it. The parameter estimates are checked for stability across the different samples. Figure 9 presents the results from this analysis.

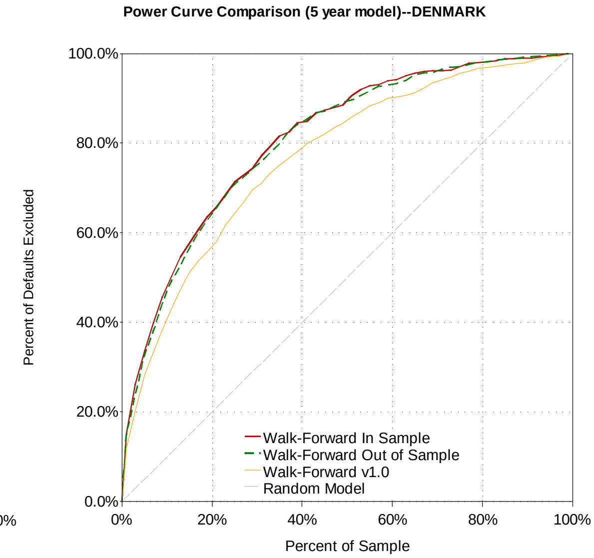
Results

Similar to the *k*-fold results, the in- and out-of-sample plots for the walk-forward results are virtually indistinguishable at both the 1- and 5-year horizons in Figure 9. The difference in AR between the in-sample and out-of-sample results is no more than 1.4% in both cases. Furthermore, RiskCalc v3.1 Denmark outperforms RiskCalc v1.0 Denmark in an out-of-time context at both the 1- and 5-year horizons.[9]

[9] The out-of-sample ARs are 71.5% and 60.2% for the 1-year and 5-year models, respectively. The 1-year out-of-sample power is 1.4 % less than the in-sample power, while 5-year out-of-sample power is 1.0 % less than the in-sample power. These out-of-sample ARs are 5.3 and 7.5 points higher than RiskCalc v1.0 Denmark for the 1- and 5-year models respectively—on the same sample.

FIGURE 9 Out-of-sample Performance (1- and 5-year) Denmark Walk-forward





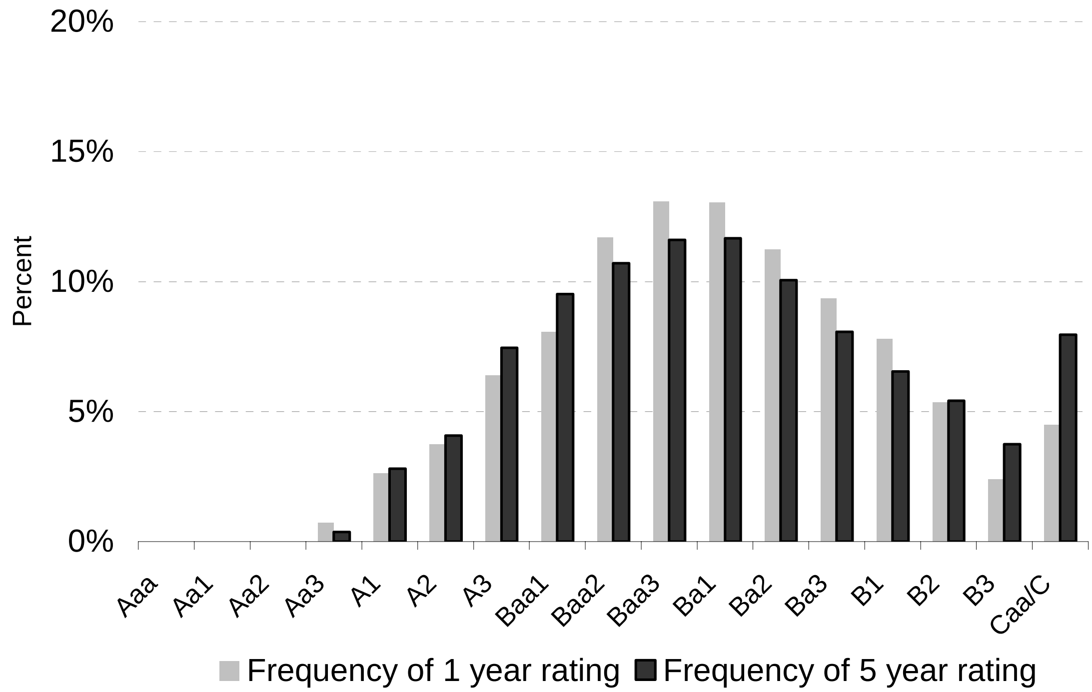
4.7 Model Calibration and Implied Ratings

To aid in the interpretation of an EDF credit measure, an EDF value is mapped to an .edf rating (an EDF-implied rating). All RiskCalc v3.1 models to date have used the same mapping. This mapping is designed so that:

- There is a large range of .edf ratings (as required for economic and regulatory applications);
- No one rating contains too many credits (as required for economic and regulatory applications);
- The distribution of the 5-year ratings is approximately the same as the distribution of 1-year ratings (for consistency with rating-based analysis applications);
- The EDF value associated with an .edf rating is approximately the same as the observed historical default rate associated with a Moody's Bond Rating (for consistency with rating-based analysis applications).

Figure 10 shows the distribution of CRD observations by rating category in the development sample (for the Credit Cycle Adjusted EDF credit measures over the full time period). Note that 14 categories between Aa3 and Caa/C are utilized and that less than 15% of the observations are in any one category. The 1-year distribution peaks at Baa3, and the 5-year peaks at Ba1. While not reported here, other research has shown that the distribution of the CCA EDF-implied ratings changes over time with the credit cycle, while the distribution of the FSO EDF-implied ratings remains relatively stable over time.

FIGURE 10 EDF-implied Ratings for the 1- and 5-year models in the development sample



5 FURTHER MODEL IMPROVEMENTS

In this section, we briefly outline some other improvements to the model. For a detailed discussion of these improvements, refer to the Technical Document.

5.1 Continuous Term Structure

The previous version of the RiskCalc model provided the user two discrete default probability estimates: a 1-year and a 5-year EDF. In this version, utilizing the two point estimates for 1- and 5-year estimates we fit a Weibull function, and thus achieve a continuous term structure of EDF values for each credit. In other words, users of RiskCalc v3.1 Denmark now can obtain EDF estimates for any point between 1 and 5 years. In addition, RiskCalc v3.1 provides EDF estimates for alternative definitions, such as the Forward EDF and the Annualized EDF (Table 17):

- **Cumulative EDF**

A cumulative EDF credit measure gives the probability of default over that time period. For example, a five-year cumulative EDF of 13.44% means that that company has a 13.44% chance of defaulting over that five-year period. The second column of Table 17 provides an example of the cumulative 1- to 5-year credit measures produced by the model.

- **Forward EDF**

The forward EDF is the probability of default between t-1 and t conditional upon survival until t-1. In other words, the 4-year Forward EDF is the probability that a firm will default between years three and four assuming the firm survived to year 3.[10] The third column of Table 17 displays the forward 1- to 5-year EDF credit measures that are derived from the cumulative EDF values.

- **Annualized EDF**

The annualized EDF credit measure is the cumulative EDF value for a given period, stated on a per year basis. For example, a company with a cumulative 5-year EDF of 13.44% would have a 5-year annualized EDF of 2.84%.[11] This means that the average default rate per year for a 13.44% cumulative default rate is 2.84%. The last column of Table 17 presents the annualized EDF credit measures for years 1 to 5 that are derived from the cumulative EDF values.

TABLE 17 Term Structure of EDF Credit Measures: An Example

EDF	Cumulative	Forward	Annualized
Year 1	4.23	4.23	4.23
Year 2	7.00	2.90	3.57
Year 3	9.37	2.55	3.23
Year 4	11.49	2.34	3.01
Year 5	13.44	2.20	2.84

5.2 New Analytical Tools: Relative Sensitivity

The RiskCalc v1.0 interface provides users an analytical tool to gauge the relative impact of each variable—as a deviation from the mean of each ratio. To equip the users of the model with further tools, we developed relative sensitivities (also known as sensitivity multiples), which exhibit the EDF sensitivity to each of the model variables at the point of evaluation. This feature is especially useful when addressing the question of identifying variables to improve the EDF value of a company.

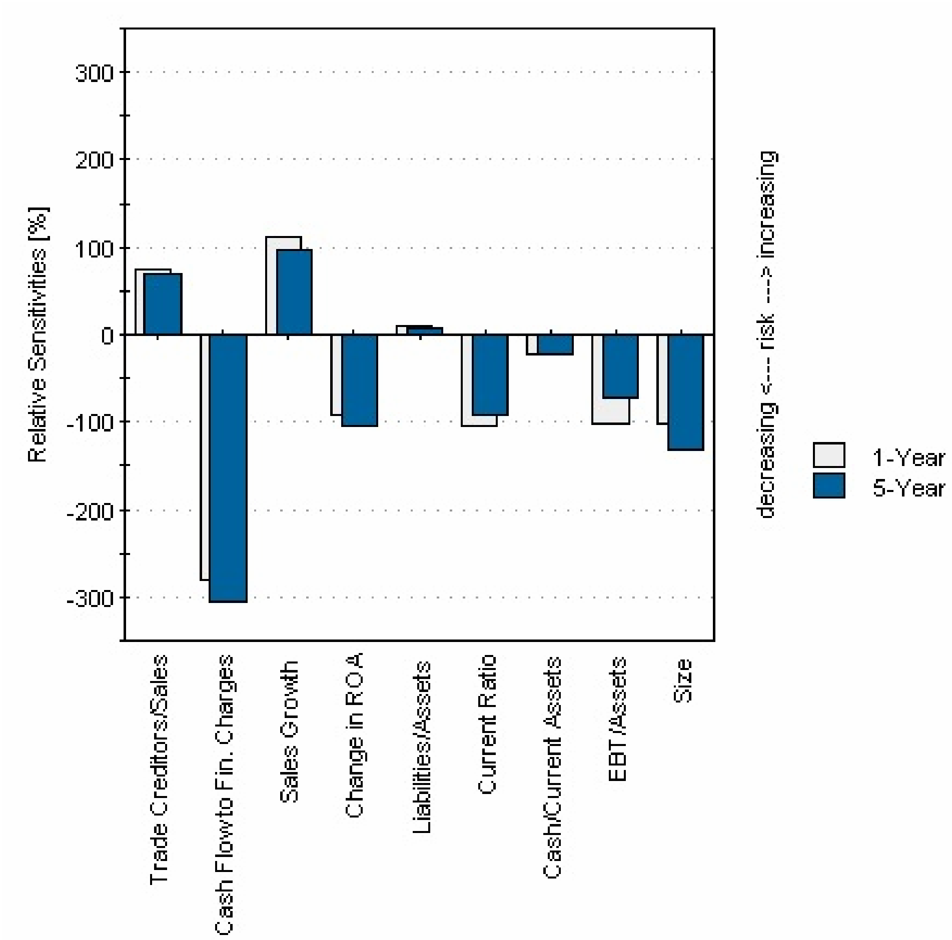
The relative sensitivity gives the impact of a small change in a variable on the EDF level of the company. It indicates which variables are most sensitive to an increase. A positive number means an increase in the variable will increase risk and a negative number indicates a decrease in risk. The percentile is the sensitivity of the variable relative to the average.

Example: A small increase in Debt Coverage Ratio (Cashflow to Financial Charges) will change the risk of the company. It is about 300% (5 year) as sensitive as the average variables (Figure 11).

[10] Specifically, $FEDF_{t-1,t} = (CEDF_t - CEDF_{t-1})/(1-CEDF_{t-1})$, where $FEDF_{t-1,t}$ is the forward EDF from years $t-1$ to t, and $CEDF_t$ is the cumulative EDF for year t.

[11] Specifically, $AEDF_t = 1 - (1-CEDF_t)^{1/t}$, where $AEDF_t$ is the annualized EDF for year t.

FIGURE 11 Relative Sensitivities



5.3 Asset Value and Volatility Calculation

One of the features of the 3.1 version of the model is that it provides an implied asset volatility. Clients of Credit Monitor and CreditEdge can use this volatility to analyze a private firm that is to go public through an IPO. After the firm is public, the public firm model should be used, however, this model requires an asset volatility that is derived from the public share price. In the 3.1 version of the model, the asset volatility of the firm is estimated using its industry and size and a methodology that is very similar to Private Firm Model. A structural model framework is then used to solve for the implied asset value from the estimated EDF credit measure, the estimated volatility, and the firm's liability structure.

6 CONCLUSION

The RiskCalc v3.1 Denmark model is based on a substantially larger database than RiskCalc v1.0 Denmark and has an additional three years of data. Improved data coverage has allowed us to refine our financial statement model and achieve a very robust prediction model of private firm default behavior.

The model is more powerful than any publicly available alternatives that we have tested. We demonstrated how the increase in power is consistent across industry sectors and size classifications as well as for different time periods. We also demonstrated how the power advantage is maintained both in- and out-of-sample.

The RiskCalc v3.1 Denmark model controls for differences in the default risk across industries in the FSO mode. In addition, in the CCA mode, it adjusts the EDF level to reflect the current stage of the credit cycle in the given industry. If default risk in a given firm's industry is high, the EDF level is adjusted upward. Likewise, when default risk is low, the EDF level is adjusted downward. This additional feature of the model increases the model power and precision and allows users to monitor their portfolios on a monthly basis.

The Moody's KMV RiskCalc v3.1 model will be useful for financial institutions seeking to implement quantitative tools for originating loans, managing portfolio risk, and meeting regulatory requirements. It also provides these institutions an objective external benchmark of the risk associated with a private firm, useful in securitizing middle-market debt. Finally, as an established benchmark, RiskCalc v3.1 enables institutions to communicate with one another about their exposures.

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