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MOODY'S KMV RISKCALC™ V3.2 CANADA

MODELINGMETHODOLOGY

ABSTRACT

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Moody's KMV RiskCalc is the Moody's KMV model for predicting private company defaults. It covers over 80% of the world's GDP, has more than 20 geographic-specific models, and is used by hundreds of institutions worldwide. While using the same underlying framework, each model reflects the domestic lending, regulatory, and accounting practices of its specific region.

In March 2009, Moody's KMV introduced its newest RiskCalc model for Canada, RiskCalc v3.2 Canada. By incorporating both market- (systematic) and company-specific (idiosyncratic) risk factors along with the newest default data reflecting the recent credit cycle development, RiskCalc v3.2 is in the forefront of modeling middle-market default risk. This modeling approach substantially increases the model's predictive powers.

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1 INTRODUCTION

The Moody's KMV RiskCalc™ v3.2 Canada model is built using the results of extensive Moody's KMV research, including the following:

- Moody's KMV RiskCalc v3.1, v1.0, and Moody's KMV Private Firm Model® (PFM)
- Moody's KMV Credit Research Database™ (CRD), the world's largest and cleanest private company default database
- Industry sector information, market information, and industry-specific default rates

RiskCalc v3.2 incorporates the structural and market-based comparables approach (used in PFM), and the localized financial statement-based approach (used in RiskCalc v1.0). This allows RiskCalc v3.2 to blend market-based (systematic) information with detailed firm-specific financial statement (idiosyncratic) information to enhance the model's predictive power. In addition, RiskCalc v3.2 incorporates the newest data (from 2003 through 2008) reflecting the recent credit environment in Canada.

1.1 RiskCalc Modes

RiskCalc v3.2 allows you to assess the risk of a private firm in two ways: Financial Statement Only (FSO) and Credit Cycle Adjusted (CCA).

The FSO mode delivers a firm's default risk based on only financial statements and sector information, adjusted to reflect differences in credit risk across industries. In this mode, the risk assessments produced by the model are relatively stable over time.

The CCA mode adjusts the default risk by taking into account the current stage of the credit cycle. The mode uses a sector-specific factor derived directly from the Moody's KMV public firm model's distance-to-default (DD). The CCA model reflects the market's current assessment of the credit cycle and is a forward-looking indicator of default.

The CCA mode is specific to the firm's sector and country and is updated monthly. The CCA mode also has the ability to stress-test Moody's KMV EDF™ (Expected Default Frequency) credit measures under different credit cycle scenarios—a requirement under Basel Capital Accord (BIS II).

1.2 Differences between RiskCalc v3.2 Canada and RiskCalc v3.1 Canada

Since the release of RiskCalc v3.1 Canada, Moody's KMV has significantly increased the size of the Canadian database and further improved its data cleansing technologies. In addition, advances in model development and testing techniques make RiskCalc v3.2 Canada more powerful and precise than its predecessor.

2 DATA DESCRIPTION

The source of the data for RiskCalc v3.2 Canada is the Moody's KMV CRD. Moody's KMV collects data from participating financial institutions, working closely with them to understand the strengths and weaknesses of the data.

2.1 Definition of Default

RiskCalc provides assistance to institutions and investors for determining the risk of default, missed payments, or other credit events. Proposals for BIS II have stimulated debates about what constitutes an appropriate definition of default. RiskCalc applies the criteria used by most of the advanced economies in the world. In model development, RiskCalc uses the local criteria for default. Accordingly, in Canada, the events which we defined as defaults include 90-days past due, bankruptcy, placement on internal non-accrual list, and write-down of the company. At the calibration stage, the model outputs are adjusted to ensure a consistent interpretation throughout the world. Specifically, the model outputs are converted into a term structure of actual default probabilities (1- through 5-year EDF credit measures).

2.2 Data Exclusions

Excluded Companies

The goal of the RiskCalc model is to provide an EDF credit measure for private Canadian companies in the middle market. The firms and industries covered in the model must have similar default characteristics. To create the most powerful model for Canadian middle-market companies, companies that did not reflect the typical company in this market were eliminated. The following types of companies are not included in the data:

- Small Companies—Companies with Real Total Assets less than $100,000 (in 2004 CAD) are not reflective of typical middle-market companies and are excluded from the database.
- Financial Institutions—The balance sheets of financial institutions (banks, insurance companies, and investment companies) exhibit higher leverage than the typical private firm. The regulation and capital requirements of these institutions make them dissimilar to the typical middle-market company. Therefore, they are excluded from the database.
- Real Estate Development Companies—The annual accounts of real estate development and investment companies provide only a partial description of the dynamics of these firms and, therefore, their likelihood to default. This is because their financial health often hinges on a particular development.[1]
- Public Sector and Non-profit Institutions—Government run companies' default risks are influenced by the states' or municipalities' unwillingness to allow them to fail. As a result, their financial results are not comparable to other private firms. Not-for-profit financial ratios are different from for-profit firms, particularly with regard to variables relating to net income.

Excluded Financial Statements

The financial statements of smaller companies can be less accurate and lower quality than those of larger companies. The financial statements in the CRD are cleaned to eliminate highly suspect financial statements. Plausibility checks of financial statements are conducted, such as Assets not equal to Liabilities plus Net Worth, and financial statements covering a period of less than twelve months. If errors are detected, those statements are excluded from the analysis.

2.3 Descriptive Statistics of the Data

Overview of the Data

The source of data for RiskCalc v3.2 Canada is the Moody's KMV CRD. Moody's KMV collects data from both participating institutions and data vendors. Table 1 summarizes the total data used in the development, validation, and calibration of the RiskCalc v3.2 Canada model. The number of financial statements, firms, and defaults utilized in v3.2 is considerably larger than v3.1. There has been more than a 118% increase in the number of unique firms, and more than a 186% increase in the number of financial statements. In addition, there has been more than a 150% increase in the number of defaults. Figure 1 presents the distribution of Canadian financial statements and defaults by year.

[1] The success of many types of project finance firms depends largely on the outcome of a particular project. We recommend using separate models for such firms. This characteristic is explicitly recognized by the Basel Accord.

TABLE 1 Canadian Private Firm Sample Data

Canadian Private Firms	RiskCalc v3.1 Canada	RiskCalc v3.2 Canada	Change
Financial statements	44,000 +	126,000 +	• 186%
Unique number of firms	11,000 +	24,000 +	• 118%
Defaults	600 +	1500 +	• 150%
Time period	1989–2002	1993–2007	+ 5 years

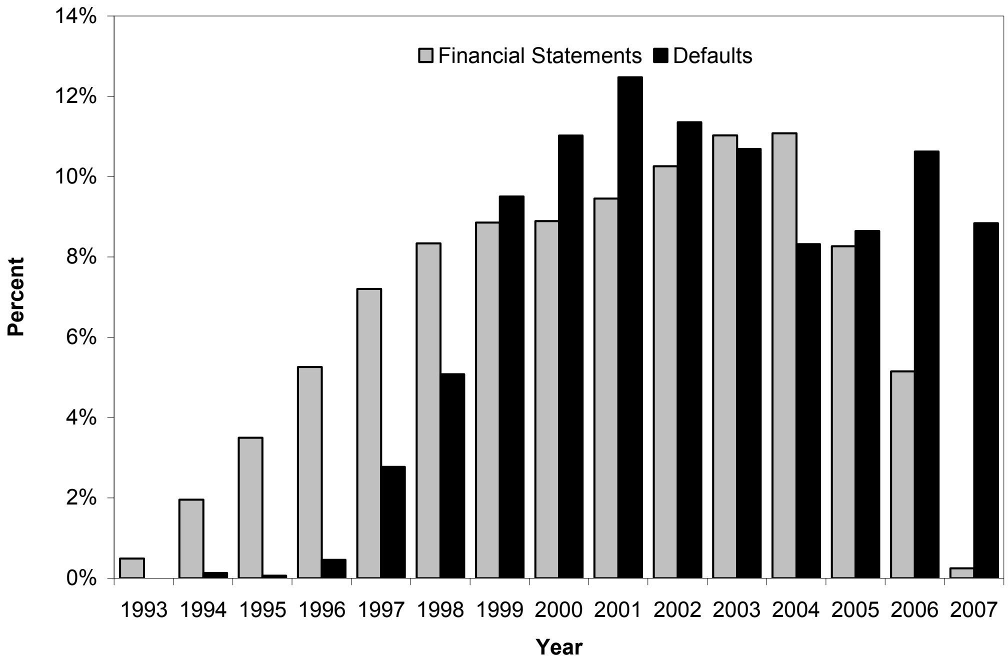

FIGURE 1 Distribution of Canadian Financial Statements and Defaults

Robustness of the Data

In building a model, potential database weaknesses need to be examined. Not only does the database need to cover a large number of firms and defaults, but the defaults also need to be distributed among industries and company types covered. For example, if the database has significant numbers of small firms, or firms in one particular industry without sufficient defaults in those areas, the model may not be a good default predictor. The CRD used in developing the RiskCalc models addresses both of these issues.

Figure 2 presents the distributions of Canadian firms by industry and the proportion of defaults in each industry. Trade is the largest sector with about 24% of the firms and 20% of the defaults in the sample. Figure 3 presents the distributions by the size of firms measured as Total Assets in 2004 Canadian dollars. These figures demonstrate how the proportion of defaults in any one industry group or size group is comparable to the proportion of firms in these groupings. The size distribution shows that about 37% of the firms hold assets less than 1 million CAD.

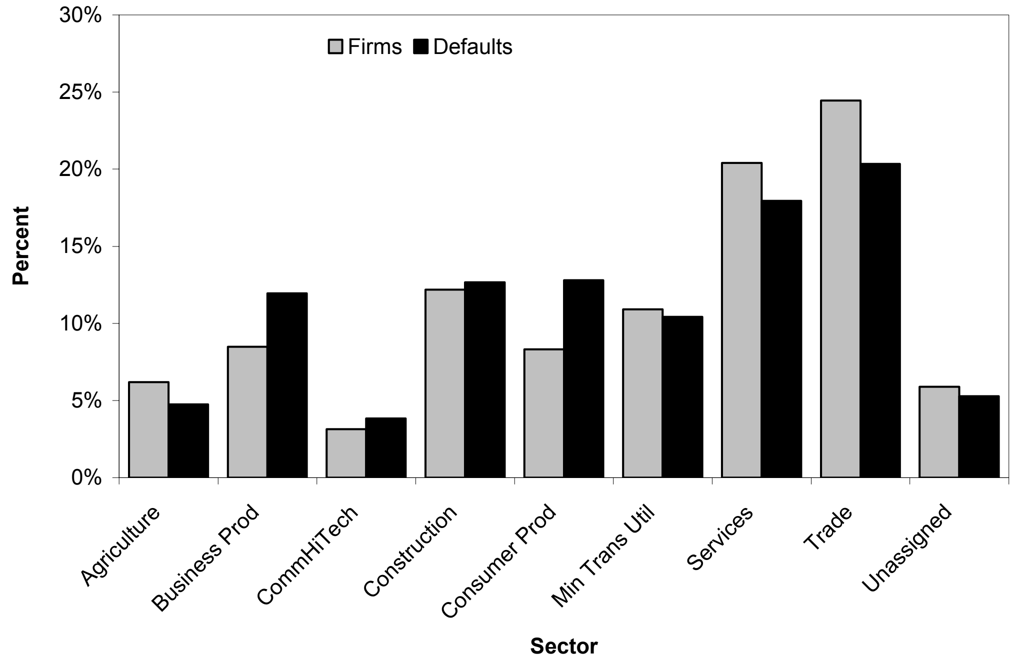

FIGURE 2 Distribution of Canadian Defaults and Firms by Industry

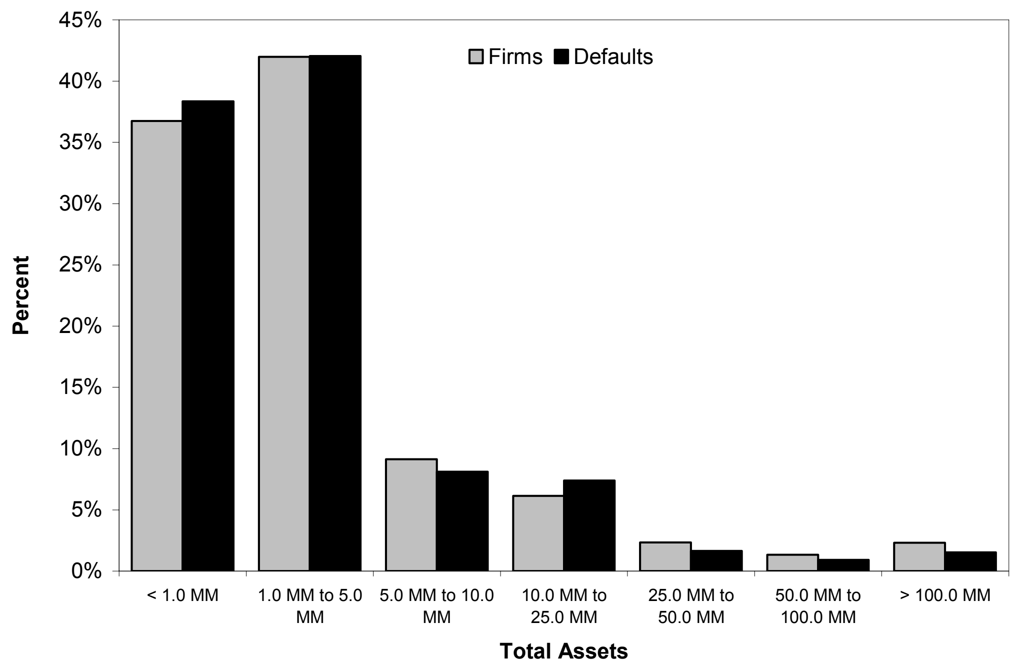

FIGURE 3 Distribution of Canadian Defaults and Firms by Size

2.4 Central Default Tendency

Because most companies do not default, companies that do default are relatively rare and thus more valuable in building a default prediction model. Much of the lack in default data comes from the data storage issues within financial institutions (e.g., defaulting companies being purged from the system after troubles begin), not all defaults being captured, or other sample errors. Also, if the date of default is uncertain, the financial statement associated with the firm may be excluded from model development, depending on the severity of the problem. This can result in a sample that has lower default rates than what actually occurs in the general population. If the underlying sample is not representative, then it needs to be adjusted for the true central default tendency (CDT). When default definitions used in the data sample understate the defaulting population, the CDT can be used to realign the default rates.

The estimate of long-run aggregate probabilities of default (i.e., CDT) is important as an anchor for a model. The best estimation of default probability is a ratio that reflects the number of obligors that defaulted in one year compared with the total obligors at the beginning of that year. Often these types of data are not available.

The estimate of the central default tendency for Canada is based on several sources.

- Loan loss provision data examined by the Organization for Economic Co-operation and Development (OECD).
- Provisioning data gathered from financial statements of large Canadian banks.
- Confirmation of the CDT exceeding the default rates observed in our development sample.

The multiple sources of external data led us to an estimate of 1.7% as the CDT figure for the 1-year model.

Calculating a 5-year Central Default Tendency

There is a lack of publicly-available data for direct calculation of the CDT of a cumulative 5-year default probability. Based on extensive Moody's KMV research, a 5-year cumulative default tendency is derived from the 1-year estimate. This research, combined with the information provided by the CRD, shows that the 5-year cumulative default rate is, on average, four times the level of the 1-year default rate. Therefore, 6.8% is used as the CDT for the 5-year model.

Central Default Tendency in FSO and CCA Modes

In FSO mode, the central default tendency remains constant over time. In CCA mode, the central default tendency is equal to the central default tendency of the FSO mode when the effects of the credit cycle are neutral. When the forward-looking prediction of the credit cycle indicates increasing default risk, the central default tendency of the CCA mode will be larger; when the effects of the credit cycle indicate reducing default risk, the central default tendency will be smaller.

3 MODEL COMPONENTS

The RiskCalc v3.2 model incorporates various components to determine the EDF credit measure. The inputs to the model include a selection of the financial ratios, transforms of those ratios, the inclusion of industry information, and the credit cycle adjustment.

The development of a RiskCalc model involves the following steps:

1. Choosing a limited number of financial statement variables for the model from a list of possible variables.[2]
2. Transforming the variables into interim probabilities of default using non-parametric techniques.
3. Estimating the weightings of the financial statement variables using a probit model, combined with industry variables.
4. Creating a (non-parametric) final transform that converts the probit model score into an actual EDF credit measure.

[2] These variables are often ratios, but not always. For example, one measure of profitability is Net Income to Total Assets, which is a ratio, and one measure of size is Inflation-adjusted Total Assets, which is not a ratio.

In FSO mode, the models are based on the following functional form.

$$FSO\ EDF = F\left(\Phi\left(\sum_{i=1}^{N}\beta_i T_i(x_i) + \sum_{j=1}^{K}\gamma_j I_j\right)\right) \tag{1}$$

where $x_1,\ldots,x_N$ are the input ratios; $I_1,\ldots,I_K$ are indicator variables for each of the industry classifications (if applicable); β and γ are estimated coefficients; Φ is the cumulative normal distribution; *F* and $T_1,\ldots,T_N$ are non-parametric transforms; and *FSO EDF* is the financial-statement-only EDF credit measure.[3] The *T*s are the transforms of each financial statement variable, which capture the non-linear impacts of financial ratios on the default likelihood as is shown in Figure 4 and discussed in detail later in the document. *F* is the final transform (i.e., the final mapping). The final transform captures the empirical relationship between the probit model score and actual default probabilities. The final transform is described as calibrating the model score to an actual EDF credit measure. The difference between the FSO EDF and the CCA EDF credit measure is that in CCA mode the final transform is adjusted to reflect our assessment of the current stage of the credit cycle, while in FSO mode it remains constant.

3.1 Financial Statement Variables

Selecting the Variables

Our variable selection process starts with a long list of possible financial statement variables addressing various risk factors. The working list of ratios is divided into groups that represent different underlying concepts regarding a firm's financial status (Table 2). A model is then built with at least one variable per group. When it is possible to increase model performance and maintain model robustness, several variables from each group will be used in the model.

We ask the following questions when deciding which variables to include in the final model:

1. Is the variable readily available?
2. Are the definitions of the inputs to the variable unambiguous?
3. Is the meaning of the variable intuitive?
4. Does the variable predict default activity?
5. Is the variable not highly correlated with other variables in the model?

The set of variables chosen for RiskCalc v3.2 Canada is similar to v3.1. "Cash Flow to Current Liabilities" from v3.1 Profitability category has been changed to "Cash Flow to Interest Expense" for v3.2 Profitability category. "Cash Flow to Interest Expense" is a more representative measure of a firm's debt coverage. Table 3 presents the variables used in the final version of RiskCalc v3.2 Canada.

[3] By non-parametric, we mean that the $T(x_i)$ is a continuous function of *x* not requiring a specification of a specific closed (i.e., parametric) functional form. We estimate these transforms using a variety of local regression and density estimation techniques.

TABLE 2 Groupings of Financial Statement Ratios

Activity ratios include inventories to sales and accounts receivable to sales. These ratios may measure the extent to which a firm has a substantial portion of assets in accounts that may be of subjective value. For example, a firm with large inventories may not be selling its products and may have to write off these inventories. → A large stock of inventories relative to sales increases the probability of default; other activity ratios have different relationships to default.
Debt Coverage is the ratio of cash flow to interest payments or some other measure of liabilities. → High debt coverage reduces the probability of default.
Growth variables are typically the change in Return on Assets (ROA) and sales growth. These variables measure the stability of a firm's performance. → Growth variables behave like a double-edged sword: both rapid growth and rapid decline (negative growth) tend to increase a firm's default probability.
Leverage ratios include liabilities to assets and long-term debt to assets. → High leverage increases the probability of default.
Liquidity variables include pure cash or cash and marketable securities to assets, the current ratio, and the quick ratio. These variables measure the extent to which the firm has liquid assets relative to the size of its assets or liabilities. → High liquidity reduces the probability of default.
Profitability ratios include net income, net income less extraordinary items, profit before tax, and operating profit in the numerator; and total assets, tangible assets, fixed assets and sales in the denominator. → High profitability reduces the probability of default.
Size variables include sales and total assets. These variables are normally deflated to a specific base year to ensure comparability (e.g., total assets are measured in 2004 Canadian dollars). → Large firms default less often.

TABLE 3 Financial Statement Variables in RiskCalc v3.2 Canada

Category	Definition
Activity	Inventory to Sales Change in AR Turnover: AR Turnover (t) – AR Turnover (t-1)[4] Current Liabilities to Sales
Debt Coverage	Cash Flow to Interest Expense[5]
Growth	Sales Growth: Net Sales(t) / Net Sales(t-1) – 1
Leverage	Retained Earnings to Current Liabilities Leverage Ratio: Long Term Debt / (Long Term Debt + Net Worth)
Liquidity	Cash and Marketable Securities to Assets
Profitability	ROA[6] Change in Return on Assets: ROA (t) – ROA (t-1)
Size	Real Total Assets

[4] AR Turnover is defined as the following: Accounts Receivable / Net Sales.

[5] Cash Flow is defined as the following: EBITDA + [Accounts Payable (*t*) - Accounts Payable (*t*-1)] - [Accounts Receivable (*t*) - Accounts Receivable (*t*-1)] - [Total Inventory (*t*) - Total Inventory (*t*-1)].

[6] ROA is defined as the following: Net Income / Total Assets.

Variable Transforms

After the variables are selected, they are transformed into a preliminary EDF value. Figure 4 presents the transformations used in the model. The horizontal axis gives the percentile score of the ratio, and the vertical axis gives the default probability of that ratio in isolation (univariate). The percentile score represents the percent of the database that had a ratio below that of the company (e.g., if ROA is in the 90th percentile, then 90% of the sample had a lower ROA than that firm).

The shape of the transformation indicates how significantly a change in level impacts the EDF value. If the slope of the transform is steep, a small change will have a larger impact on risk than if the slope were flat.

- For the **Activity** group, the ratios are Inventory to Sales, Current Liabilities to Sales, and Change in AR Turnover (Figure 4). Inventory to Sales and Current Liabilities to Sales are both upward-sloping, indicating that firms with large inventory or current liabilities relative to sales have higher default probabilities. Change in AR Turnover is U-shaped, indicating that large increases or decreases in accounts receivable turnover are associated with higher default probabilities, while stable accounts receivable turnover is associated with lower default probabilities.
- For the **Debt Coverage** group, the transform for Cash Flow to Interest Expense is downward-sloping (Figure 4). The slope of the transform decreases as debt coverage increases. This indicates that firms with large Cash Flow relative to Interest Expense have lower default probabilities.
- For the **Growth** group, the transform for Sales Growth is U-shaped (Figure 4). This indicates that large increases or decreases in sales are associated with higher default probabilities, while stable sales year-upon-year decreases the probability of default. The actual shape indicates that large increases in sales increase default probabilities by a larger amount than large decreases in sales.
- For the **Leverage** group, the ratios are Retained Earnings to Current Liabilities and Long Term Debt to Long Term Debt plus Net Worth (Figure 4). The transform for Retained Earnings to Current Liabilities is downward-sloping. The transform for Long Term Debt to Long Term Debt plus Net Worth is upward-sloping. Large values of Retained Earnings to Current Liabilities decrease default probabilities, while Long Term Debt to Long Term Debt plus Net Worth increase default probabilities.
- For the **Liquidity** group, the transform for Cash and Marketable Securities to Assets is downward-sloping (Figure 4). Firms with large cash holdings are associated with lower default probabilities.
- For the **Profitability** group, the ratios are ROA and Change in ROA (Figure 4). ROA's transform is downward-sloping; however, the slope approaches zero as ROA becomes large. Such a transform indicates that more profitable firms have lower default probabilities, but the impact diminishes as ROA increases. Change in ROA is U-shaped, indicating that large increases or decreases in ROA increase the default likelihood. The actual transform shape indicates that large decreases in ROA increase the likelihood of default by a larger amount than large increases in ROA.
- For the **Size** group, the transform for Real Total Assets is downward-sloping (Figure 4). The slope of the transform decreases as size increases. This indicates that larger firms have lower default probabilities, but the impact of size on default probabilities diminishes as firm size increases.

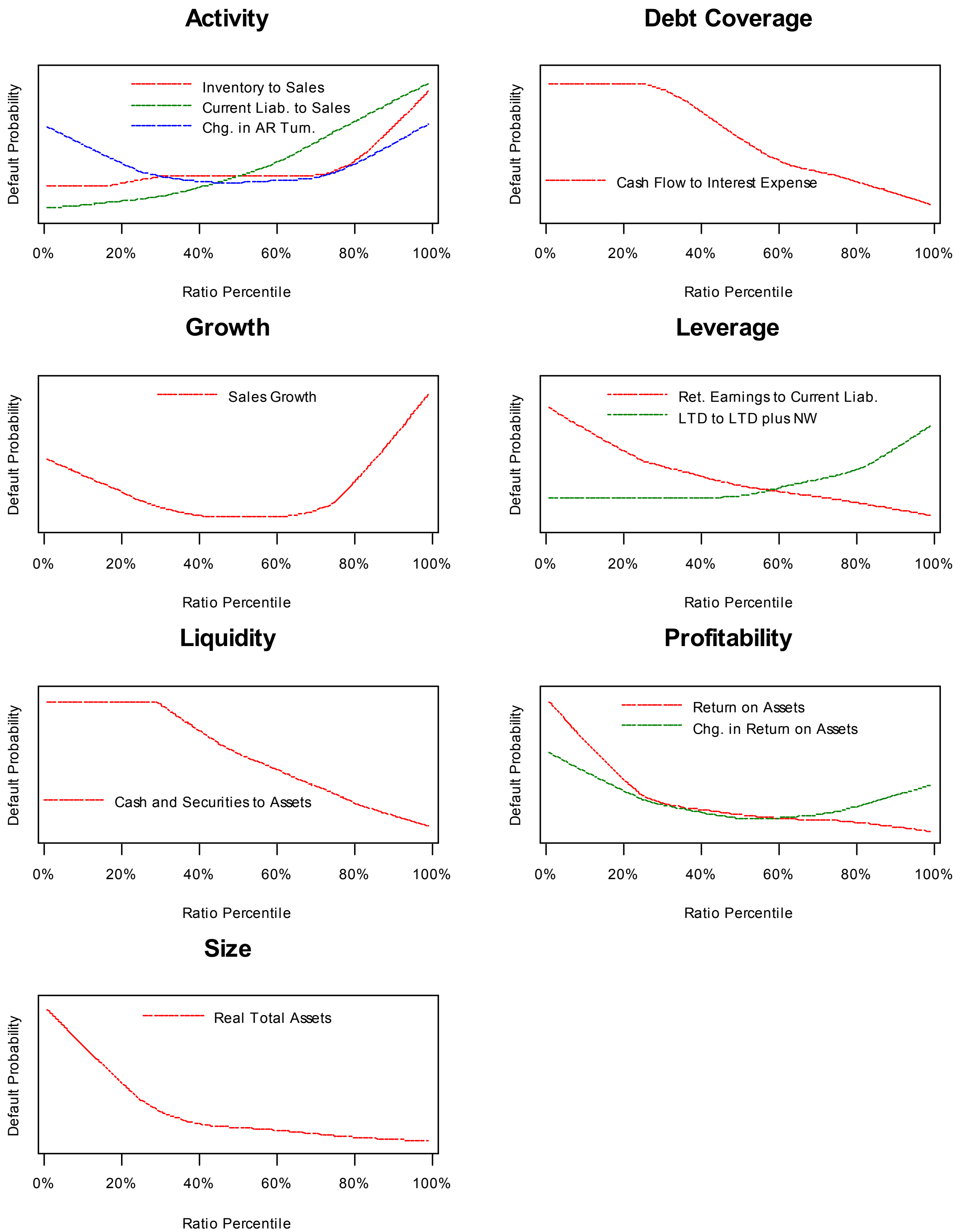

FIGURE 4 Transformations of Financial Statement Variables in RiskCalc v3.2 Canada

3.2 Model Weights

Importance

The relative value of each variable used in calculating an EDF credit measure is important in understanding a company's risk. The non-linear nature of the model makes the weight of the variables more difficult to determine, because the actual impact on the risk depends on the coefficient, the transformation shape, and the percentile ranking of the company. The model weights, therefore, are calculated based on the average EDF value for the transformation and its standard deviation. A variable with a flat transformation could have a low weight, even if the coefficient is large (Figure 4).

Calculation of Weights

To calculate the weighting of a variable, the EDF credit measure is computed for a theoretical firm with all its variables at the average transformation values. The variables are then increased one at a time by one standard deviation. The EDF level change for each variable (in absolute value) is computed and added together. The relative weight of each variable is then calculated as the EDF level change for that variable as a percent of the sum of EDF level changes across all variables. This gives the variable with the largest impact on the EDF level the largest weight, and the variable with the smallest impact on the EDF level the smallest weight. Because the weights are a percentage of the total change in EDF levels, they sum to 100%. The weight of each category is the sum of the weights of each variable in that category.

RiskCalc v3.2 has been "re-weighted" according to the new data, reflecting the new credit environment that is beyond the coverage of v3.1. Table 4 presents the weights in RiskCalc v3.2 Canada. The most important categories are Profitability and Capital Structure. Relative to v3.1, the importance of Size and Profitability increased while Activity and Capital Structure somewhat decreased.

TABLE 4 Risk Drivers in RiskCalc v3.2 Canada

RiskCalc v3.1 Canada		RiskCalc v3.2 Canada	
Risk Drivers	**Weight**	**Risk Drivers**	**Weight**
Capital Structure Retained Earnings to Current Liabilities LTD to LTD plus NW	25%	Capital Structure Retained Earnings to Current Liabilities LTD to LTD plus NW	21%
Liquidity Cash to Assets	16%	Liquidity Cash to Assets	18%
Activity Inventory to Sales Current Liabilities to Sales Change in AR Turnover	29%	Activity Inventory to Sales Current Liabilities to Sales Change in AR Turnover	20%
Growth Sales Growth	6%	Growth Sales Growth	5%
Size Real Total Assets	4%	Size Real Total Assets	10%
Profitability ROA Change in ROA Cash Flow to Current Liabilities	20%	Profitability ROA Change in ROA Cash Flow to Interest Expense[7]	26%

[7] Cash Flow to Interest Expense is categorized in this manner to compare weights with v3.1; otherwise, it is categorized as Debt Coverage in v3.2.

3.3 Industry Adjustments

While the variables included in the RiskCalc model explain most of the risk factors, the relative importance of the variables can be different among industries. In the FSO mode of RiskCalc v3.2 Canada, the EDF value is adjusted for industry effects. Table 5 presents the increase in model power and accuracy from introducing industry controls into the FSO model. Both the power and the accuracy of the EDF credit measure increase, as measured by the Accuracy Ratio (AR) and the gain in log likelihood. A large gain in likelihood indicates that the industry controls are especially important in producing an accurate EDF credit measure. Table 5 presents the average FSO EDF value by industry for the validation sample.

TABLE 5 Increase in Model Power and Accuracy from Introducing Industry Controls

FSO Mode	1-year Model		5-year Model	
	Accuracy Ratio	Relative Increase in Log Likelihood	Accuracy Ratio	Relative Increase in Log Likelihood
Without Industry Controls	63.3%	---	48.0%	---
With Industry Controls	64.1%	32.2***	49.2%	99.9***

*** Indicates a p-value of less than 0.1% via a ^{2} test

In this table, and hereafter, AR is the measure of the model's ability to rank order credits. Increases in log likelihood measure the extent to which the model's EDF values match observed default rates.[8] In Table 6, the values show the combined impact of the industry adjustment and the average levels of each ratio. The combination of the two determines the average EDF credit measure for a company.

TABLE 6 Average EDF Credit Measure in April 2007 by Sector

Sector	Average 1-year EDF	Average 5-year EDF
Agriculture	1.4%	6.1%
Business Products	0.9%	6.8%
Telecommunications and High Tech	1.7%	3.6%
Construction	0.6%	5.3%
Consumer Products	1.1%	9.5%
Mining, Transportation, Utilities, and Natural Resources	2.6%	5.7%
Services	1.4%	5.1%
Trade	1.1%	5.1%

3.4 Credit Cycle Adjustment

EDF credit measures are impacted not only by the financials of a company, but also by the general credit cycle in the economy. To capture this effect, RiskCalc v3.2 Canada includes a credit cycle adjustment (CCA) factor. The CCA is designed to incorporate the current credit cycle into the estimate of private firm default risk.

[8] For further details, see Dwyer and Stein (2004), Technical Document on RiskCalc v3.1 Methodology.

Selecting an Adjustment Factor

The RiskCalc v3.2 model uses the DD calculation from the Moody's KMV public firm model. This measure is specifically designed to be a forward-looking indicator of default risk. It extracts signals of default risk from the stock market performance of individual firms.[9] This measure was chosen because it is available for a large universe of industries and it has been extensively validated.[10]

If the DD factor for public firms in an industry indicates a level of risk above the historical average for that industry, then the private firms' EDF values in that industry are adjusted upward. Conversely, if the level of risk is below the historical average for that industry, then the private firms' EDF values are adjusted downward. When the credit cycle adjustment factor is neutral, the CCA EDF value coincides with the FSO EDF value.

Adjustment Factor Used in the Model

For the Canadian model, the DD factor is based on an aggregation of all public North American firms in the industry. In the event that a firm cannot be associated with a specific industry, the model uses a credit cycle adjustment that is based on an aggregation of all public North American firms.

Figure 5 presents the DD factor based on all public North American firms and contrasts that to the speculative grade default rate as measured by Moody's Investors Service default studies.[11] For both recessions, the speculative default rate increases in advance of the recession so that a risk indicator that is coincident with the business cycle will not predict increases in risk. The DD factor anticipates both the recession and the increase in defaults measured by the speculative default rate. Therefore, it is a forward-looking measure of default risk in an industry.

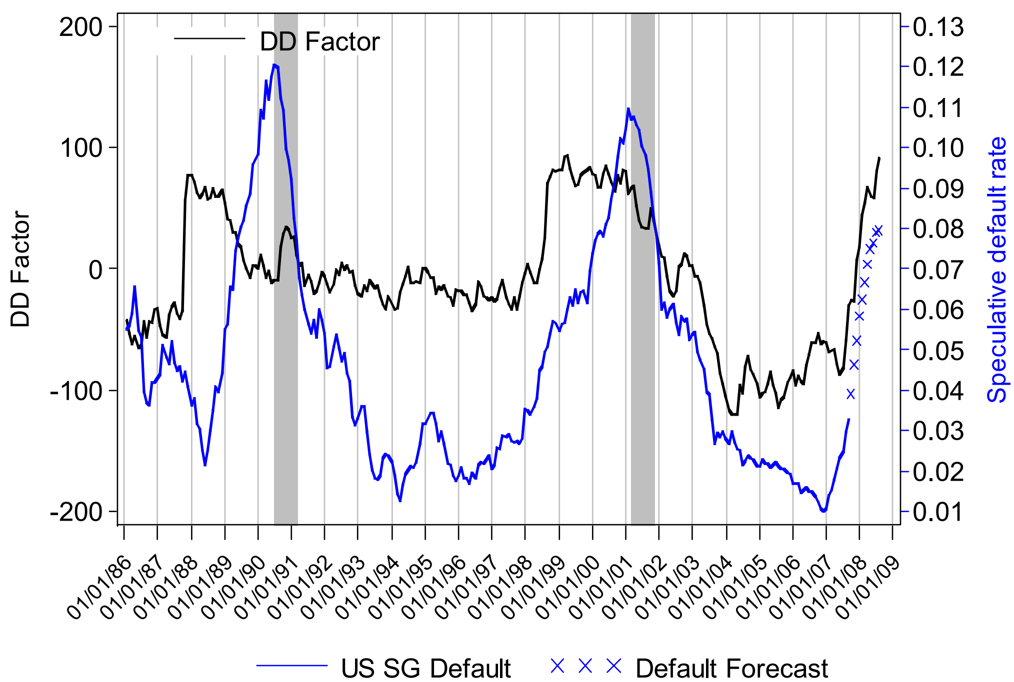

FIGURE 5 North American DD Factor and the Speculative Grade Default Rate: 1986–2008

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[9] cf. Bohn and Crosbie, 2003.

[10] cf. Dwyer and Korablev, 2007.

[11] cf. Hamilton and Varma, 2003.

Figure 5 presents the DD factor (solid black line) against the historical Speculative Grade Bond Default Index (solid blue line, the blue cross marks indicate forecasted values). The gray vertical bars indicate periods of recession as defined by the National Bureau of Economic Research (NBER). Large values of DD factor provided early warnings of increased default rates for both recessions.

4 VALIDATION RESULTS

After a model is developed, it must be proven effective in predicting defaults. In this section, testing results are presented on the model's ranking power (i.e., the model's ability to sort credits from worst to best).

The tests need to check not only the model effectiveness, but also its robustness and how well it works on data outside the sample. Both *k*-Fold and walk-forward analyses were performed for out-of-sample testing. The results of the testing showed that the model is uniformly more powerful than other models across different sectors, size classifications, and time periods.

4.1 Increase in Overall Model Power and Accuracy

Table 7 presents the in-sample overall measures of power for RiskCalc v3.2 Canada versus alternative models for the validation sample.

In CCA mode, the model's performance improves by 3.7% at the 1-year horizon, and 4.9% at the 5-year horizon compared to RiskCalc v3.1 Canada.[12] We present the ARs for the CCA model relative to RiskCalc v3.1 and Z-score for the power tests across different sectors, size classifications, and time periods.

Table 7 also contains p-values for the statistical test to display how the accuracy ratio from the RiskCalc v3.2 FSO model and the benchmark is less than or equal to zero. A p-value of less than .05 indicates we can reject the hypothesis that the difference in the accuracy ratios is less than or equal to zero with 95% confidence.[13]

Relative to other available alternatives, the results were more dramatic. The new RiskCalc v3.2 model, in FSO mode, outperformed the Z-score model by 15.5% at the 1-year horizon and 18.3% at the 5-year horizon.[14]

TABLE 7 Power Enhancements of the RiskCalc v3.2 Canada Model—Validation Sample

Mode	1-year Model		5-year Model	
	Accuracy Ratio	p-value for the Increase over v3.2 Accuracy Ratio	Accuracy Ratio	p-value for the Increase over v3.2 Accuracy Ratio
RiskCalc v3.2	63.7%	---	51.3%	---
RiskCalc v3.1	60.0%	<.0001	46.4%	<.0001
Z-score	48.2%	<.0001	33.0%	<.0001

[12] The corresponding accuracy ratios are 64.1% (FSO) vs. 60.4% (RiskCalc v3.1) for the 1-year horizon and 49.2% (FSO) vs. 43.7% (RiskCalc v3.1) for the 5-year horizon.
[13] For more details on the computation of the p-value, see Hood (2007), Comparing the Performance of Two Models: Analytical and Bootstrapping Methods.
[14] cf. Altman, Hartzell, and Peck, 1995.

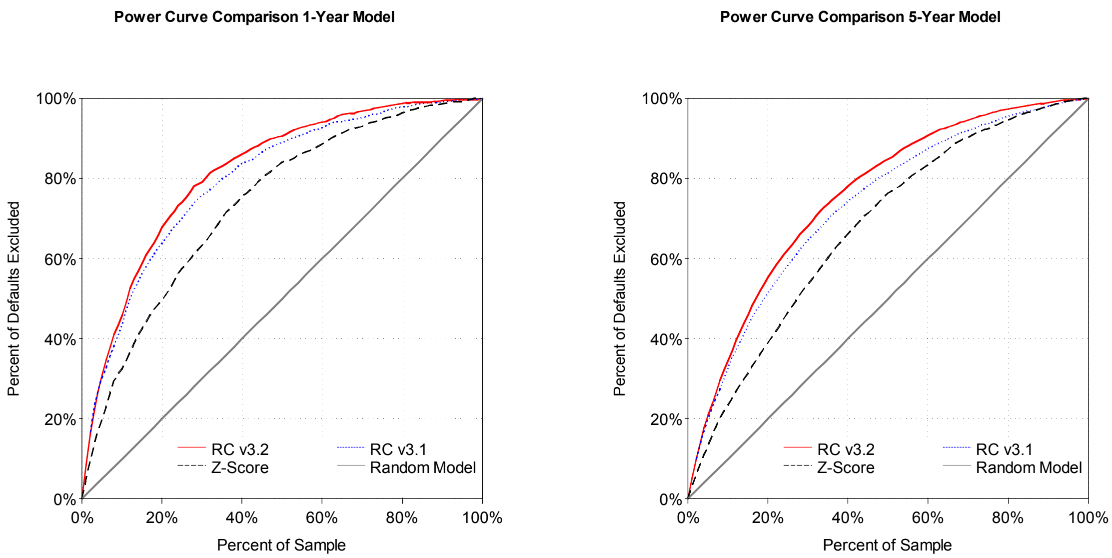


FIGURE 6 Power of Alternative Models (1- and 5-year)—Canada Validation Sample

Figure 6 presents the cumulative accuracy profiles for the 1- and 5-year models corresponding to Table 7. The power improvements are uniformly significant across different regions of the distribution relative to RiskCalc v3.1.

4.2 Correlations and Variance Inflation Factors

To ensure model robustness, the model must be tested for excessive multicollinearity, which occurs if a number of the variables used in the model are highly correlated. Excessive multicollinearity can cause instability in parameter estimates. To check for this issue, the correlation coefficients in Table 8 for the financial statement ratios in the model and the variance inflation factors (VIF) in Table 9 are computed on the transformed variables displayed in Figure 4.[15]

Model Results

This section shows the results of the model, after being tested for excessive multicollinearity. Table 8 displays the correlations among the transformed input factors. Table 9 displays the VIF levels.

[15] For further definitions and technical discussion of the testing procedures in “Validation Results” on page 17, refer to the Technical Document.

TABLE 8 Correlations Among the Transformed Input Factors (Spearman Rank)

	Inventory to Sales	Current Liabilities to Sales	Cash Flow to Interest Expense	Sales Growth	Change in Return on Assets	Change in AR Turnover	Retained Earnings to Current Liabilities	Long Term Debt to Long Term Debt plus Net Worth	Cash to Assets	Return on Assets	Real Total Assets
Inventory to Sales	1.00										
Current Liabilities to Sales	0.23	1.00									
Cash Flow to Interest Expense	0.21	0.26	1.00								
Sales Growth	-0.01	0.11	0.04	1.00							
Change in Return on Assets	-0.06	0.01	0.01	0.17	1.00						
Change in AR Turnover	-0.02	0.18	0.00	0.28	0.12	1.00					
Retained Earnings to Current Liabilities	0.04	0.36	0.31	0.09	0.07	0.03	1.00				
Long Term Debt to Long Term Debt plus Net Worth	-0.07	0.20	0.26	0.01	-0.01	-0.06	0.32	1.00			
Cash to Assets	0.19	0.16	0.31	-0.02	-0.02	0.00	0.17	0.15	1.00		
Return on Assets	0.11	0.28	0.44	0.06	0.28	0.05	0.38	0.15	0.13	1.00	
Real Total Assets	-0.06	-0.05	0.02	0.06	0.18	0.01	0.08	0.06	-0.03	0.07	1.00

The VIF levels in Table 9 for the financial statement variables represent how much of the variation in one independent variable can be explained by all the other independent variables in the model. The correlation coefficient, however, measures only the relationships between two variables. The VIF levels are all below 2, indicating that the collinearity between the variables is low.[16] The two ratios with the highest correlation are Cash Flow to Interest Expense and Return on Assets in Table 8.

[16] As Woolridge (2000) shows, VIF is inversely related to the tolerance value (1- R^2), such that a VIF of 10 corresponds to a tolerance value of 0.10. Clearly, any threshold is somewhat arbitrary and depends on the sample size. Nevertheless, if any of the R^2 values are greater than 0.75 (so that VIF is greater than 4.0), we would typically suspect that multicollinearity could be a problem. If any of the R^2 values are greater than 0.90 (so that VIF is greater than 10) we then conclude that multicollinearity is likely to be a serious problem.

TABLE 9 Variance Inflation Factors

Variable	VIF
Return on Assets	1.51
Cash Flow to Interest Expense	1.49
Retained Earnings to Current Liabilities	1.40
Current Liabilities to Sales	1.35
Inventory to Sales	1.24
Long Term Debt to Long Term Debt plus Net Worth	1.24
Change in AR Turnover	1.21
Change in Return on Assets	1.18
Cash to Assets	1.15
Sales Growth	1.13
Real Total Assets	1.07

4.3 Power Performance by Industry and Size Groups

It is important to test the power of a model not only overall, but also among different industry segments and firm sizes.

Table 10 and Table 11 present the power comparisons by sector for the 1-year and 5-year models, respectively. RiskCalc v3.2 Canada outperforms both RiskCalc v3.1 Canada and Z-score in almost all sectors.[17] The highest power in the 1-year horizon (Table 10) is found in Mining, Transportation, Utilities & Natural Resources (69.0%), while the lowest is found in Agriculture (45.6%). At the 5-year horizon (Table 11), the highest power is in Services (53.0%), and the lowest is in Agriculture (41.7%).

TABLE 10 Power by Industry 1-year Canada Model—Validation Sample

	Percentage of Defaults	AR v3.2	AR v3.1	v3.2–v3.1 p-value	AR Z-score
Agriculture	3%	45.6%	52.3%	0.0206	50.0%
Business Products	13%	58.6%	56.9%	0.0706	48.4%
Telecommunications & High Tech	4%	58.8%	54.2%	0.0383	39.9%
Construction	13%	60.7%	56.1%	0.0003	50.8%
Consumer Products	14%	59.6%	58.2%	0.1677	47.7%
Mining, Transportation, Utilities & Natural Resources	11%	69.0%	67.8%	0.3111	54.6%
Services	20%	65.7%	62.5%	0.0016	48.8%
Trade	22%	62.0%	59.7%	0.0212	47.3%

[17] The only exception is in Agriculture at the 1-year and 5-year horizon, but the difference is not statistically meaningful at the 5-year horizon. Agriculture is the sector with the smallest number of defaults.

TABLE 11 Power by Industry 5-year Canada Model—Validation Sample

	Percentage of Defaults	AR v3.2	AR v3.1	v3.2–v3.1 p-value	AR Z-score
Agriculture	3%	41.7%	44.5%	0.4257	26.4%
Business Products	13%	45.3%	44.2%	0.3370	30.2%
Telecommunications & High Tech	4%	47.9%	46.2%	0.4602	38.3%
Construction	14%	48.1%	45.0%	0.0218	35.7%
Consumer Products	14%	50.4%	48.0%	0.1093	34.4%
Mining, Transportation, Utilities & Natural Resources	11%	52.0%	47.0%	0.0026	33.3%
Services	19%	53.0%	51.6%	0.2201	34.6%
Trade	22%	49.9%	46.3%	0.0032	31.4%

Table 12 and Table 13 present the power comparisons by firm size (Total Assets in 2004 CAD) for the 1-year and 5-year models, respectively. RiskCalc v3.2 Canada outperforms both RiskCalc v3.1 Canada and Z-score in all size groups. The highest power in the 1-year is found in the 1 mm to 5 mm range, while highest power in the 5-year is found in the 5 mm to 10 mm range. The lowest powers in the 1-year and 5-year are both found in the less than 1 mm range.

TABLE 12 Power by Size (Total Assets in 2004 CAD) 1-year Canada Model—Validation Sample

Range	Percentage of Defaults	AR v3.2	AR v3.1	v3.2–v3.1 p-value	AR Z-score
< $ 1 MM	33%	57.6%	56.1%	0.0527	47.4%
$ 1 MM to 5 MM	44%	65.5%	61.7%	<.0001	48.4%
$ 5 MM to 10 MM	11%	61.4%	54.8%	0.0003	39.3%
> $ 10 MM	12%	60.4%	55.9%	0.0029	40.6%

TABLE 13 Power by Size (Total Assets in 2004 CAD) 5-year Canada Model—Validation Sample

Range	Percentage of Defaults	AR v3.2	AR v3.1	v3.2–v3.1 p-value	AR Z-score
< $ 1 MM	35%	42.8%	41.7%	0.2254	33.7%
$ 1 MM to 5 MM	43%	54.2%	50.5%	<.0001	37.4%
$ 5 MM to 10 MM	11%	59.0%	52.5%	0.0039	32.4%
> $ 10 MM	11%	54.0%	48.2%	0.0015	26.3%

4.4 Power Performance over Time

Because models are implemented at various points in a business cycle by design, power tests by year were conducted to examine whether the model performance is excessively time-dependent.

Table 14 and Table 15 present the results from this analysis at the 1- and 5-year horizons, respectively. The AR of RiskCalc v3.2 Canada is compared with RiskCalc v3.1 Canada and Z-score for each year. RiskCalc v3.2 Canada outperforms both RiskCalc v3.1 Canada and Z-score from 1997 onward.[18]

[18] The new model working better on the 1997–2006 sample and worse on the 1995–2006 is not surprising. The pre-1997 data represented a larger portion of the v3.1 development data set than the v3.2 development data set, and consequently the model weights for v3.1 are "more optimized" for that time period. The model weights for v3.2 are "more optimized" for the recent time period.

TABLE 14 Power over Time: 1-year Horizon, Canada Model—Validation Sample

Year	Percentage of Defaults	AR v3.2	AR v3.1	v3.2–v3.1 p-value	AR Z-score
1995	2.2%	59.6%	69.5%	0.0013	50.8%
1996	5.9%	74.8%	77.5%	0.0585	63.0%
1997	8.4%	71.6%	69.4%	0.0322	55.9%
1998	11.3%	64.8%	62.0%	0.0198	54.9%
1999	14.0%	58.3%	55.5%	0.0179	50.1%
2000	12.3%	58.7%	54.8%	0.0006	41.8%
2001	10.2%	61.6%	54.3%	<.0001	40.6%
2002	8.4%	58.9%	50.2%	<.0001	40.8%
2003	8.6%	53.3%	47.4%	0.0003	32.8%
2004	9.6%	53.7%	46.7%	<.0001	28.3%
2005	7.3%	58.0%	49.9%	<.0001	34.7%
2006	1.8%	56.7%	48.9%	0.0001	41.4%

TABLE 15 Power over Time: 5-year Horizon, Canada Model—Validation Sample

Year	Percentage of Defaults	AR v3.2	AR v3.1	v3.2–v3.1 p-value	AR Z-score
1995	6.3%	56.9%	57.4%	0.7381	43.7%
1996	9.1%	59.3%	57.3%	0.0552	45.4%
1997	11.7%	55.1%	52.1%	0.0019	42.4%
1998	12.6%	51.0%	47.5%	0.0017	41.5%
1999	12.7%	50.1%	46.3%	0.0008	38.1%
2000	10.7%	49.2%	44.1%	<.0001	33.2%
2001	10.0%	50.8%	43.7%	<.0001	31.7%
2002	9.1%	46.0%	37.7%	<.0001	27.9%
2003	7.8%	45.2%	37.5%	<.0001	23.3%
2004	6.0%	48.7%	40.1%	<.0001	22.6%
2005	3.3%	56.8%	48.8%	0.0001	32.7%
2006	0.7%	56.9%	52.6%	0.0344	41.4%

4.5 Out-of-sample Testing: *k*-Fold Tests

The model exhibits a high degree of power in distinguishing good credits from bad ones (Table 7), but whether this power is attributable to the overall model effectiveness or the impact of a particular subsample also needs to be tested. A standard test for evaluating this is the *k*-Fold test, which divides the defaulting and non-defaulting companies into *k* equally-sized segments. This yields *k* equally-sized observed subsamples that exhibit the identical overall default rate and are temporally and cross-sectionally independent. The model is then run on *k*-1 subsamples and these parameter estimates are used to score the *k*-th subsample. This procedure is repeated for all possible combinations, and the *k* scored out-of-sample subsamples are put together to calculate an accuracy ratio on this combined data set.

Table 16 summarizes the *k*-Fold test results (with *k*=5) for the development sample. The reported figures are the accuracy ratios by the corresponding sample and time horizons. The out-of-sample model consistently outperforms RiskCalc v3.1 Canada. Figure 7 presents the cumulative accuracy profiles associated with the overall out-of-sample results against the in-sample results. The model performance is maintained both in- and out-of-sample in the *k*-Fold analysis.

TABLE 16 RiskCalc v3.2 Canada *k*-Fold Test Results

	Out-of-sample AR		RiskCalc v3.1	
	1-year AR	5-year AR	1-year AR	5-year AR
Subsample 1	60.9%	53.9%	56.5%	50.4%
Subsample 2	61.9%	54.3%	57.4%	50.7%
Subsample 3	60.1%	52.7%	57.2%	48.3%
Subsample 4	62.9%	51.7%	59.1%	48.2%
Subsample 5	60.1%	51.8%	58.4%	49.7%
k-fold Overall	63.7%	49.6%	60.1%	46.6%
In-sample AR	64.1%	49.8%	---	---

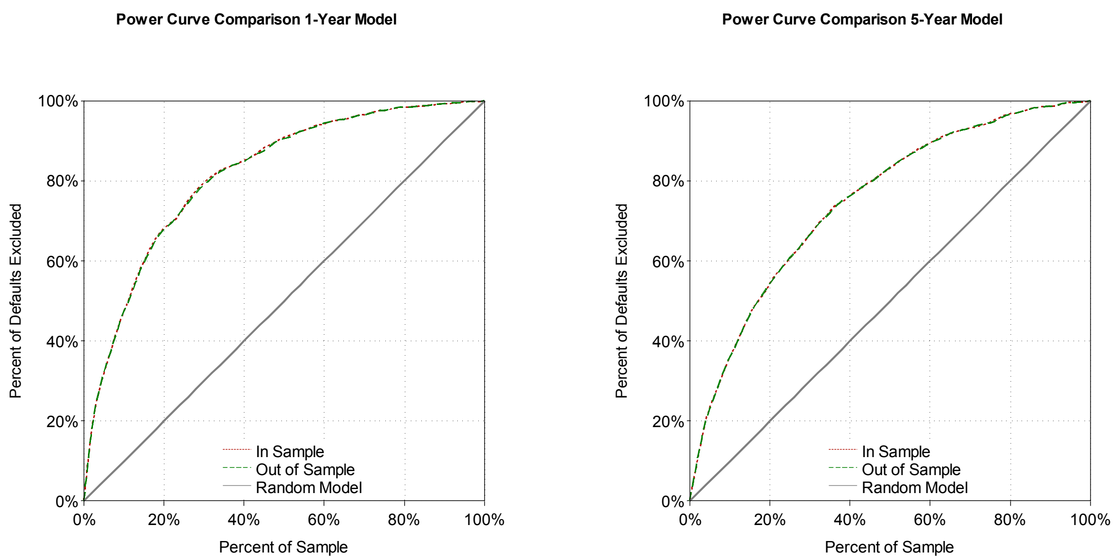


FIGURE 7 Out-of-sample Performance (1- and 5-year) Canada *k*-Fold

The *k*-Fold testing does not control for time-dependence. Each of the *k* subsamples contains data from all periods. As a result, if there were a particularly high period of default rates, this would be included in each of the *k* samples. Such testing does not give a true sense of how the model would have performed during those volatile periods, because the model is estimated with full information on those time periods.

Results

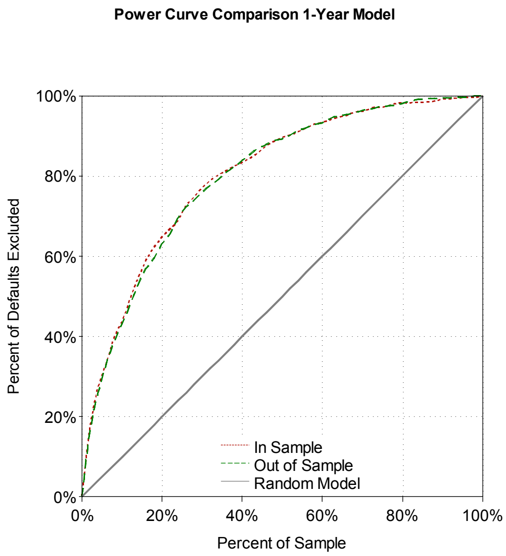
The in- and out-of-sample plots are virtually indistinguishable at both the 1- and 5-year horizons in Figure 7. The difference in AR between the overall in-sample and out-of-sample results is not larger than 50 basis points for the 1-year and not larger than 20 basis points for the 5-year. Furthermore, RiskCalc v3.2 Canada outperforms RiskCalc v3.1 Canada in an out-of-sample context at both the 1- and 5-year horizons (Table 16).

4.6 Walk-forward Tests

An alternative out-of-sample test developed by Moody's KMV is a walk-forward test, which is designed along similar lines as the *k*-Fold test, except that it controls for the effects of time. The model is estimated up to a certain year and the parameter estimates are then used to score the observations in the next year. These model scores are out-of-time. The model is re-estimated including one more year of data. The analysis is then repeated for the next year, and continued until the end of the sample. These out-of-sample out-of-time scores are combined into a single prediction set, so that the accuracy ratio and the power curve can be calculated for the combined set. The out-of-sample accuracy ratio is then compared to the corresponding in-sample accuracy ratio and power curve.

No data from a future period is used in fitting the model, and only data from future periods is used for testing it. The parameter estimates are checked for stability across the different samples. Figure 8 presents the results from this analysis.

The difference in ARs between the in-sample and out-of-sample results is 0.8% for the 1-year and -0.7% for the 5-year.[19]



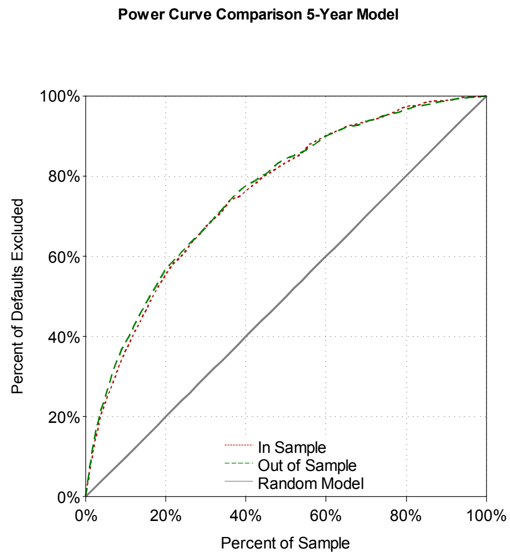


FIGURE 8 Out-of-sample Performance (1- and 5-year) Canada Walk-forward

4.7 Model Calibration and Implied Ratings

As many credit analysts are trained to differentiate credit quality in terms of letter grades, the model maps the EDF credit measures to an EDF-implied rating utilizing Moody's bond default studies. All RiskCalc v3.2 models to date have used the same mapping. This mapping is designed with the following considerations:

- There is a large range of EDF-implied ratings (as required for economic and regulatory applications).
- No one rating contains too many credits (as required for economic and regulatory applications).
- The distribution of the 5-year ratings is approximately the same as the distribution of 1-year ratings (for consistency with rating-based analysis applications).
- The EDF value associated with an EDF rating is approximately the same as the observed historical default rate associated with a Moody's bond rating (for consistency with rating-based analysis applications).

Figure 9 shows the distribution of CRD observations by rating category in the development sample. Note that 13 categories between A1 and Caa/C are utilized, and that less than 15% of the observations are in any one category. The distributions peak at Ba1 for both the 1-year and the 5-year ratings. While not reported here, other research has shown

[19] The out-of-sample AR is 60.8% for the 1-year model and 52.3% for the 5-year model. The out-of-sample AR is 5.2% higher than RiskCalc v3.1 Canada for the 1-year model and 4.1% for the 5-year model.

that the distribution of the CCA EDF-implied ratings changes over time with the credit cycle, while the distribution of the FSO EDF-implied ratings remains relatively stable over time.

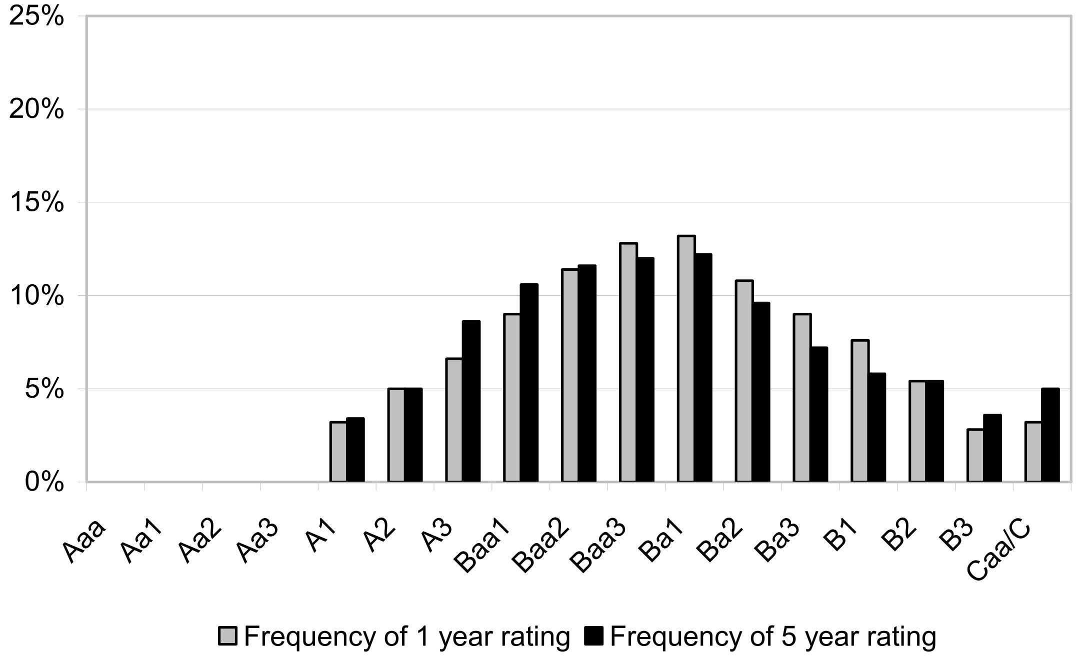


FIGURE 9 EDF-implied Ratings for the 1- and 5-year Models in the Development

5 FURTHER MODEL IMPROVEMENTS

This section outlines some other improvements to the model.[20]

5.1 Continuous Term Structure

The previous version of the RiskCalc model provided two discrete default probability estimates: a 1-year and a 5-year EDF credit measure. In this version, utilizing the two-point estimates for 1- and 5-year estimates fits a Weibull function, and thus achieves a continuous term structure of EDF values for each credit. In other words, users of RiskCalc v3.2 Canada can now obtain EDF values for any point between one and five years. In addition, RiskCalc v3.2 provides EDF values for alternative definitions, such as the forward EDF value and the annualized EDF value (Table 17).

Cumulative EDF Credit Measures

A cumulative EDF credit measure gives the probability of default over that time period. For example, a 5-year cumulative EDF credit measure of 13.44% means that a company has a 13.44% chance of defaulting over that five year period. The second column of Table 17 provides an example of the cumulative 1- to 5-year credit measures produced by the model.

[20] For a detailed discussion of these improvements, refer to the Technical Document.

Forward EDF Credit Measures

The forward EDF credit measure is the probability of default between t-1 and t conditional upon survival until t-1. In other words, the 4-year forward EDF credit measure is the probability that a firm will default between years three and four assuming the firm survived to year three.[21] The third column of Table 17 displays the forward 1- to 5-year EDF credit measures that are derived from the cumulative EDF values.

Annualized EDF Credit Measures

The annualized EDF credit measure is the cumulative EDF value for a given period, stated on a per-year basis. For example, a company with a cumulative 5-year EDF value of 13.44% would have a 5-year annualized EDF value of 2.84%.[22] This means that the average default rate per year for a 13.44% cumulative default rate is 2.84%. The last column of Table 17 presents the annualized EDF credit measures for years one to five. These credit measures are derived from the cumulative EDF values.

TABLE 17 Term Structure of EDF Credit Measures: An Example

EDF	Cumulative	Forward	Annualized
Year 1	4.23	4.23	4.23
Year 2	7.00	2.90	3.57
Year 3	9.37	2.55	3.23
Year 4	11.49	2.34	3.01
Year 5	13.44	2.20	2.84

5.2 New Analytical Tools: Relative Sensitivity

The RiskCalc v3.2 application provides an analytical tool to gauge the relative impact of each variable—as a deviation from the mean of each ratio. Relative sensitivities (also known as sensitivity multiples) exhibit the EDF sensitivity to each of the model variables at the point of evaluation. This feature is especially useful when addressing the topic of identifying variables to improve the EDF value of a company.

The relative sensitivity gives the impact of a small change in a variable on the EDF level of the company. It indicates which variables are most sensitive to an increase. A positive number means an increase in the variable will increase risk, and a negative number will decrease risk. The percentile is the sensitivity of the variable relative to the average.

For example, a small increase in ratio for Current Liabilities to Sales increases the risk of the company. It is about 170% (5-year) as sensitive as the average variable (Figure 10).

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[21] Specifically, $FEDF_{t-1,t} = (CEDF_t - CEDF_{t-1})/(1-CEDF_{t-1})$, where $FEDF_{t-1},t$ is the forward EDF from years t-1 to t, and $CEDF_t$ is the cumulative EDF for year t.

[22] Specifically, $AEDF_t = 1 - (1-CEDF_t)^{1/t}$, where $AEDF_t$ is the annualized EDF for year t.

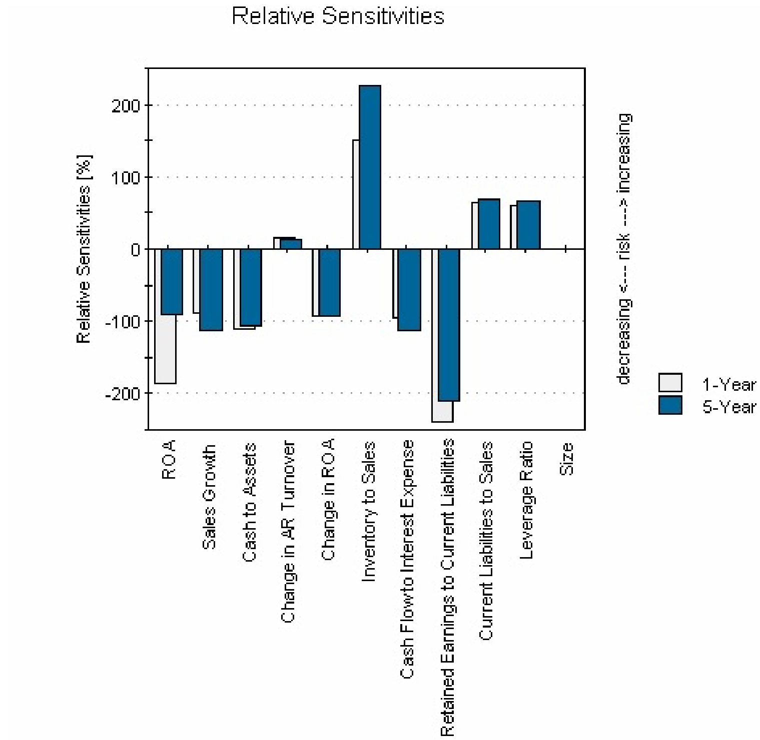

FIGURE 10 Relative Sensitivities for the RiskCalc v3.2 Model

5.3 Asset Value and Volatility Calculation

One useful feature of the PFM model is that it provides an implied asset value and an estimate of the firm's volatility. For example, clients can use this feature to analyze IPO activities. To address this, as an add-on to the model, the asset volatility of the firm is estimated using its industry and size, and a methodology that is similar to the PFM model. A structural model framework is used to solve for the implied asset value from the estimated EDF value, the estimated volatility, and the firm's liability structure.

5.4 Percentile Map

The percentile map feature allows users to quickly isolate the problematic ratios for a given company. As shown in Figure 11, each horizontal bar represents a ratio that is labeled on the left (e.g., Return on Assets). The column on the right displays the actual value of the ratio. The percentage number within the horizontal bar graph represents the percentile of the ratio within the development sample (e.g., 80.7% of the development sample had a Return on Assets less than 12.50%). The shading represents the risk level associated with the ratio: green is low risk, red is high risk, and grey is neutral risk. The variables shaded red to green represent "good" ratios for which high values lower risk, while the variables shaded green to red represent "bad" ratios for which high values increase risk. Change in ROA, Change in AR Turnover, and Sales Growth are shaded red to green to red. This demonstrates that both high and low values for Change in AR Turnover indicate high risk, while moderate Change in AR Turnover indicates low risk. For this hypothetical firm, the problematic ratios are Cash Flow to Interest Expense, Size, Leverage Ratio, and Inventory to Sales.

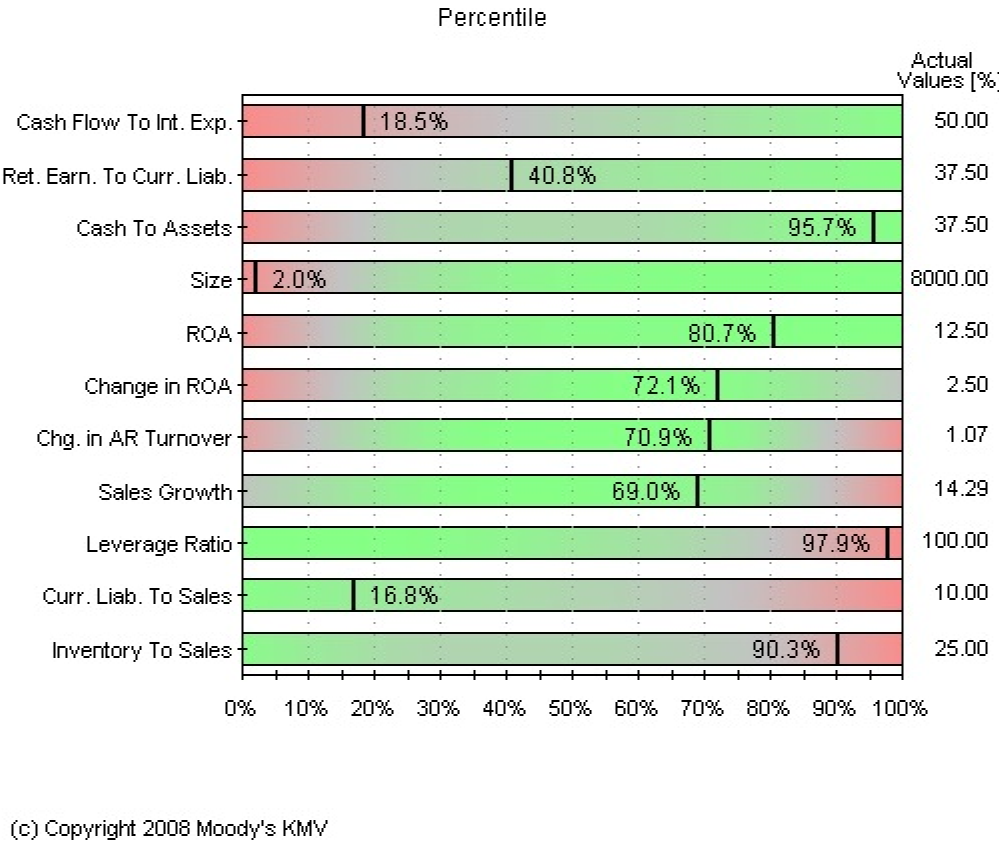

FIGURE 11 Percentile Graph for the RiskCalc v3.2 Model

6 CONCLUSION

The RiskCalc v3.2 Canada model is based on a substantially larger database than RiskCalc v3.1 Canada, with over twice as many defaults, a 118% increase in the number of firms, and a 186% increase in the number of financial statements. Furthermore, it has an additional five years of data. Improved data coverage allowed us to refine our financial statement model and achieve a very robust prediction model of private firm default behavior.

The model is more powerful than any publicly available alternatives that we have tested. We demonstrated how the increase in power is consistent across industry sectors and size classifications, as well as for different time periods. We also demonstrated how the power advantage is maintained both in- and out-of-sample.

The RiskCalc v3.2 Canada model controls for differences in the default risk across industries in the FSO mode. In addition, in the CCA mode, it adjusts the EDF level to reflect the current stage of the credit cycle in the given industry. If default risk in a given firm's industry is high, the EDF level is adjusted upward. Likewise, when default risk is low, the EDF level is adjusted downward. This additional feature of the model increases the model power and precision and allows users to monitor their portfolios on a monthly basis.

This model is useful for financial institutions seeking to implement quantitative tools for originating loans, managing portfolio risk, and meeting regulatory requirements. It also provides these institutions an objective external benchmark of the risk associated with a private firm, useful in securitizing middle-market debt. Finally, as an established benchmark, RiskCalc v3.2 enables institutions to communicate with one another about their exposures.

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