LEVEL AND RANK ORDER VALIDATION OF RISKCALC V3.1 UNITED STATES

MODELINGMETHODOLOGY

AUTHORS

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ABSTRACT

In this paper, we validate the Moody's KMV RiskCalc v3.1 United States private firm default model. We show that the EDF™ (Expected Default Frequency) produced by the model continues to rank order risk effectively by providing substantial discriminatory power across multiple cuts of the data.

We also validate the EDF level produced by the model. For most development datasets used to build private firm default models, direct level validation is not possible due to multiple issues encountered when working with private firm data. The U.S. is an exception, because we have been collecting loan accounting system data for the past nine years from multiple banks. This data enables direct measurement of the default rate on a cohort of active borrowers. Using the loan accounting system data, we find that the EDF level is consistent with the realized default rates observed over the last nine years.

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1 INTRODUCTION

Practitioners are interested in how models perform. One measure of performance is a model's ability to rank order credits (i.e., separate the risky firms from the safe firms). In practice, a model's ability to rank order is measured through a cumulative accuracy profile and its corresponding statistics. Practitioners who use models to monitor their exposures and create watch lists may find this aspect particularly interesting. A model that can effectively rank order risk allows a risk manager to allocate efficiently monitoring resources.

Another performance measure is the extent to which the level of the probability of default (PD) is "correct." We ask the following question: is a 1-in-36 measure of default risk actually a 1-in-36 chance of default? This question is difficult to address, and the methodologies to address it are still being developed. Nevertheless, clients using a model to measure Value at Risk or to estimate reserves will be very interested in this measure of performance. The probability of default level is a key determinant of both required regulatory and required economic capital. If the PD level is too high, then capital requirements are unnecessarily high and the return on equity will suffer. If the PD level is too low, then capital requirements are too low and there may be unexpected capital restrictions, forcing distressed sales, or worse, bank failures. We anticipate regulators would be interested in how a bank justifies the PD level in its models. Also, if a bank is securitizing middle market loans into a CLO, getting the PD level right is critical to the performance of the structure.

In this paper, we present the results of a study validating the EDF™ (Expected Default Frequency) from Moody's KMV RiskCalc™ v3.1 United States private firm model. This widely-used model was released in 2004.[1] We show that the model continues to rank order risk effectively by providing substantial discriminatory power between defaulting firms and non-defaulting firms. The model's predictive power in a pure, out-of-sample context is comparable to its performance prior to its release.[2] Not surprisingly, the model performs better when used on borrowers with more timely financial statement information.

For more than nine years, the Moody's KMV Credit Research Database™ (CRD) has been collecting loan accounting system information from multiple financial institutions in North America. This information allows us to track an actual cohort of borrowers at a bank and determine whether or not they are actually making payments on their loans. Such data is essential in order to assess if the PD level produced by a model is consistent with the realized default rate.

To assess if the level of a PD model is correct, we must define default. For this paper, we define default as either accounts that are 90 days past due and accompanied by a non-pass grade, or accounts designated as charge-offs, non-accruals, doubtful loans, or loss. For these default types, it is highly likely that the bank will suffer an actual economic loss because of the default. In our view, this default definition is the most appropriate for this population. We view this definition as consistent with how default is defined in the rating agency default studies, in other Moody's KMV products, and in the Basel Accord.

For completeness, we plan to test the model against an extended definition of default that includes near-defaults. We define a near-default as a *Substandard Account* that does not progress to a more severe default type. Risk managers should be interested in model performance with respect to this definition as well, as it indicates the power of discriminating firms at the earliest signs of default. We will show the model maintains its high level of performance when tested against these "near-defaults."

This paper is organized in the following way:

- Section 2 discusses the data issues in building private firm default models and the implications for validation.
- Section 3 describes advances in middle market databases, including refinements in the default detection methodology, which allow for more effective database construction. We also discuss the data filters used in building the datasets for this paper and provide summary statistics.
- Section 4 presents the rank order validation.

[1] cf. Dwyer and Kocagil, 2004; and Dwyer and Stein, 2005.

[2] After we release a model and clients start using it, we do not change the model—given the same inputs, the model produces the same outputs.

- Section 5 discusses level validation.
- Section 6 provides concluding remarks.
- Appendix A outlines a maximum likelihood technique used to estimate asset correlation.
- Appendix B provides the Basel definition of default as well as implementation guidance on this definition from the U.S. regulatory bodies.

2 DATA CHALLENGES AND IMPLICATIONS FOR VALIDATION

We can think of a PD at time *t* for a specific obligor as the probability of default between *t* and *t+x* given information available to the lender at time *t.* There are many ways to use financial statements when monitoring a firm's credit risk.[3] One can take a *financial statement-based* approach, where a customer's risk is assessed after receiving a financial statement. Another option would be a *calendar date-based* approach, where a specific date is chosen, and the most recently available statement at that time is assessed. In the U.S., where firms' fiscal year-ends are staggered throughout the year, banks can implement a variety of practices in this regard.[4] The financial statement-based approach evaluates a firm's credit risk once a statement is received, uses a financial statement as the observation, and defines the default flag relative to the financial statement date. The calendar date-based approach identifies firms that are actively borrowing at a specified point in time and evaluates the firm's credit risk at this time based on the most recent prior financial statement available. This latter method more naturally connects defaults to obligors and assigns defaults to the time period in which they occurred.

Information on private firms is harder to acquire than information on public firms. In some countries, companies are required to register their financial statements and their legal status with the Companies House.[5] This information is then made publicly available. In such countries, one can build a private firm default model using this information. In other countries, such information is not available, so one has to pool information from different financial institutions to build a private firm default model. One advantage of working with data provided by financial institutions rather than the Companies House is that financial institutions are typically in a better position to provide default information that is more consistent with the Basel definition of default.

In the U.S., we work with data from banks and financial institutions. Consequently, default information is extracted from a bank's payment records. Because data is provided anonymously, we cannot identify specific borrower names. Although rare, two banks may provide information from the same firm. In this case, the borrower is counted as two distinct borrowers.[6]

Private firm default models are often estimated using a financial statement-based structure. Typically, we can assume that the information contained in the financial statement would be available after a certain time interval following the end of the fiscal year. Consequently, the default window is conventionally defined as a certain window following the financial statement date (e.g., 90–455 days after the financial statement release date). Validating the level of a PD model using this database construction has several known challenges.[7]

First, a bank may have a financial statement for a firm in its systems even though the firm was not actively borrowing from the bank at that time. When banks originate a new loan, they ask for the two most recent financial statements and

[3] In addition to full year statements, we do receive a mixture of interim, preliminary, and quarterly financial statements. Nevertheless, we choose not to work with the quarterly statements as the coverage is very thin and the quality is questionable.

[4] Financial institutions frequently monitor exposures on a regular review cycle. Of course, more resources are dedicated to monitor the relatively larger exposures.

[5] There is a *Companies House* in the UK. Throughout Europe, *Companies House Data* has become somewhat of a generic term for this type of data.

[6] We have identified several pairs of financial statements provided by different banks as likely coming from the same firm. We suspect duplication because the financial statements match across several different lines items. Nevertheless, we have only found a very limited number of possible duplications in the data.

[7] In the original RiskCalc Methodology Document (Falkenstein, 2000), Section III (authored by Jim Herrity) describes in detail the challenges in linking financial statement information with default information they encountered at that time.

enter both into their systems.[8] In such cases, we will not observe the firm defaulting within 15 months of the first financial statement because the originating bank does not yet have a loan outstanding for the obligor.[9] In addition, the possibility of a firm defaulting within 15 months of the second financial statement is often limited. For example, a firm applies for a loan at the end of September 2003 with financial statements dated December 31 for both 2001 and 2002. In order to be observed defaulting within 15 months of the second financial statement, the borrower must default before April 2004, within six months of applying for the loan. The time from the application approval and funding to the time payment is ceased and the firm is classified as a defaulter can easily exceed more than six months. Consequently, we find significantly lower observed defaults on the first and second statements which may not be representative of the risk of these firms.[10]

Second, many private firm defaulters do not deliver their last financial statement. For example, suppose a firm's fiscal year ends in December and they defaulted in May 2009. They probably would have delivered their December 2007 financial statement in February or March 2008. There is a good chance, however, that they would not have delivered their December 2008 financial statement in February or March 2009, because this would have been around the time they stopped making payments on their debt. In the Moody's KMV CRD, 25% of the defaults do not have financial statements within 15 months of the default date. This type of sample selection bias also causes the sample default rate to be lower than the population default rate.

In practice, private firm default probability models are not typically calibrated to the default rate observed in the development sample, but rather to a central default tendency. The central default tendency is an estimate of the average default rate for the relevant population based on several sources of information. By collecting loan accounting system data, this technique is becoming less necessary. As a result, level validation of private firm default models is no longer solely an assessment of the reasonableness of these alternative central default tendency estimates.

3 ADVANCES IN MIDDLE MARKET DATABASES

In both the U.S. and Canada, we have been collecting loan accounting system data since 1999. This data enables us to do the following.

- Accurately count the number of obligors designated as active borrowers at a specific point in time.
- Accurately count the number of those active borrowers who defaulted during a time interval that follows that point in time.

We can use this data to accurately measure a default rate and then compare the realized default rate with a model's predictions.

3.1 Refinements in the Default Detection Methodology

In 2009, the Moody's KMV CRD refined the default detection methodology used to flag defaults in loan accounting system data. We implemented this change to make defaults more consistent with our interpretation of the Basel II default definition. Further, the new default definition allows for greater consistency in detecting defaults across banks.

The new methodology detects all types of defaults and near-defaults for all banks, over time. Here, we define a near-default as a Substandard account that does not progress to a more severe default type. Once all defaults are detected, we aggregate the data to create a single default for each borrower who has a default event of Substandard or more severe. The defaulted borrower is then assigned a date of the earliest default event and the most severe default type detected over time.[11] Table 1 lists the severity order for the old and new methodology.

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[8] Of course, the extent to which this practice is implemented varies both across and within financial institutions.

[9] The firm could have defaulted against its prior bank or other creditors, but we would not record it as such.

[10] Dwyer, 2005.

[11] For defaults, the default date is the date of the first occurrence of 90 DPD Non-Pass Rating, Non-accrual, Doubtful, Loss, Charge-off, and Bankruptcy. For defaults and near-defaults, the default date is the date of the first occurrence of Substandard 90 DPD, Non-Pass Rating, Non-accrual, Doubtful, Loss, Charge-off, and Bankruptcy.

TABLE 1 Default Type by Severity

Severity Order	New Methodology	Severity Order	Old Methodology
1	90 DPD w/ pass rating	1	90 DPD
2	Substandard	2	Substandard
3	90 DPD w/ non-pass rating	3	Non-accrual
4	Non-accrual	4	Doubtful
5	Doubtful	5	Loss
6	Loss	6	Charge-off
7	Charge-off	7	Bankruptcy
8	Bankruptcy		

The major improvement with this new default detection methodology is the identification of technical 90 days past due (90 DPD) defaults which are events documented as 90 DPD that do not progress to an economic loss. In the past, no distinction was made between a technical 90 DPD default and an actual 90 DPD default. We now make this distinction by defining a technical 90 DPD as an obligor that is 90 DPD but has a pass-grade rating, based on regulatory standards. If a borrower with a 90 DPD indicator is also assigned a non-pass regulatory rating, the default is flagged as an actual default. If the borrower is assigned a pass rating, the event is considered a technical default.

Prior to the methodology change, we suspected that 90 DPD defaults that did not progress to a more severe default type were technical defaults. These defaults had much stronger financial statements than other, more severe default types. Because we were unable to decipher between a non-technical and a technical 90 DPD default in the past, we typically ran our analysis both with and without 90 DPD defaults that did not progress to a more severe level of default. In our view, the new methodology, which identifies non-technical 90 DPD defaults, is more consistent with the Basel default definition.

Basel II defines a default as 90 days past due on any material credit obligation, or when the bank considers the loan unlikely to pay. We interpret a 90 DPD default still considered pass-grade by the bank as not being 90 days past due on a material credit obligation; our view is that if it were a material obligation, it would not be considered pass-grade. The Basel Accord includes several factors indicating that a loan could be classified as unlikely to pay. These include non-accruals, charge-offs and write-downs, distressed exchanges, and bankruptcy. We directly capture non-accruals, charge-offs, and write-downs. We capture distressed exchanges and bankruptcies to the extent that they lead the credit to be placed on non-accrual, charge-offs, and assigned a loss. We also include a doubtful asset as a default since it is considered unlikely to pay. In this paper, we treat Substandard borrowers as near-defaults. We also assess model performance when including these near-defaults.[12] The U.S. regulatory bodies do not include the Substandard classification in their definition of unlikely to pay in *Risk-Based Capital Standards: Advanced Capital Adequacy Framework—Basel II; Final Rule*.[13]

[12] A "substandard" asset is defined as "inadequately protected by the current sound worthiness and paying capacity of the obligor or by the collateral pledged, if any. Assets so classified must have a well-defined weakness, or weaknesses that jeopardize the liquidation of the debt. They are characterized by the distinct possibility that the institution will sustain some loss if the deficiencies are not corrected." This definition contrasts with an asset classified as "doubtful," which "has all the weaknesses inherent in one classified substandard with the added characteristic that the weaknesses make collection or liquidation in full, on the basis of currently known facts, conditions, and values, highly questionable and improbable."

[13] Appendix B provides both the Basel definition of default and the U.S. Agencies' guidance on the implementation of this definition.

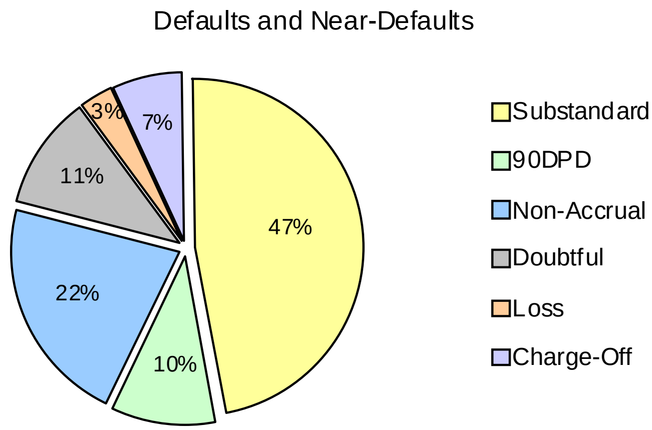

FIGURE 1 Defaults and Near-defaults

3.2 Data Construction

We construct the dataset as follows. We include each obligor active in the loan accounting system data at April 1 of each year. We then find the obligor's most recent annual financial statement available to the bank in April.[14] We use that statement to compute a credit cycle-adjusted EDF, where the credit cycle adjustment is based on a "current date" of April of that year. We then flag an observation as a default if the first default event occurs between April 1 of that year and March 31 of the next year.

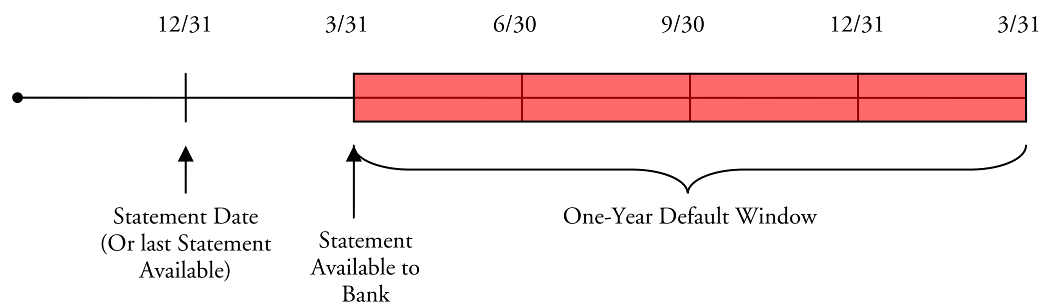

FIGURE 2 Description of Default Definition

Because this data structure is based on firms that are active borrowers at the beginning of the default window, all observations in this dataset could actually be observed defaulting during the following year. Further, we can capture all default events to the extent that they are captured in the loan accounting system data. This database construction is different from the conventional database construction for private firm default models. We call this database construction a calendar date-based database to contrast it with what a financial statement-based database. With a calendar date database construction, financial statement age will differ across firms. For most firms, the fiscal year will end in December and the most recent financial statement will be three months old. For firms whose fiscal year ends in June, however, the most recent financial statement will be nine months old on April 1. Again, a financial statement-based database is constructed so that the most recent financial statement will always be three months old. A calendar date-based

[14] In the United States, most annual financial statements are based on a December 31 fiscal year end. Our assumption is that the December 31 financial statement for the prior year is available to the bank in April. However, should an obligor's fiscal year end in January, February, or March, the financial statement would not be available to the bank by April 1 of that year.

database mimics a financial institution by reviewing its entire portfolio on April 1 of every year. A financial statement-based database mimics a financial institution by reviewing a loan whenever a new annual financial statement is received.

The two structures would coincide if: (i) all firms submitted financial statements every year, (ii) the fiscal year end was December 31 for all firms, and (iii) banks did not backfill financial statements into their systems when new borrowers applied for loans. These conditions are not met in most private firm databases.

To highlight the implications of the two different database constructions, Figure 3 presents the default rates implied by the two different database structures: the financial statement-based database and the calendar date-based database. We present the default rates by borrower tenure to highlight how these different approaches can also lead to different conclusions about the risk of subgroups. The borrower tenure is measured by the number of financial statements available in the respective database for the borrower, up to and including the given date. For example, the 0.45% default rate for a borrower tenure of 2 means that 0.45% of the borrower's default within [90,455] days of their second financial statement in the financial statement-based dataset. Likewise, the 1.65% default rate for a borrower tenure of 2 means that 1.65% defaulted within [90,455] days of their second financial statement in the calendar date-based database. The reason for the difference: many firms would not have actually been borrowing from the bank at the time of their first few financial statements in the financial statement-based dataset.

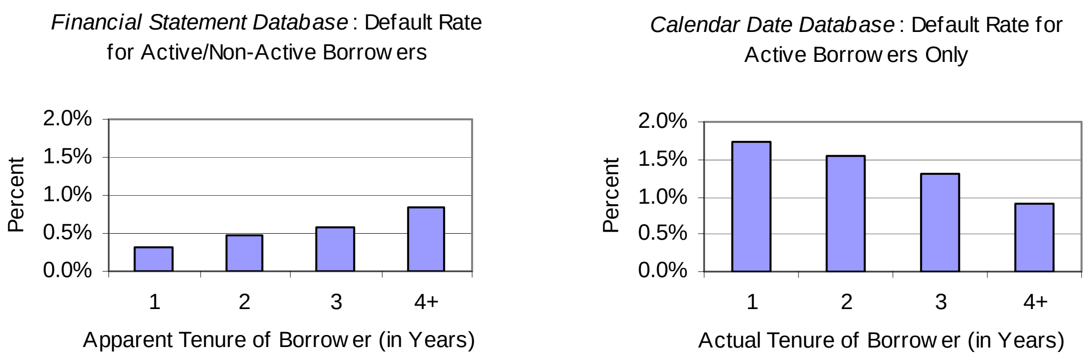

FIGURE 3 Comparison of Database Constructions

As expected, the financial statement-based database provides a lower estimate of historical default rates since it contains financial statements from non-active borrowers. This difference is pronounced for the first and second observations for each borrower. This database construction also misrepresents the historical default pattern for new versus mature borrowers. It implies that new borrowers are much safer when, in reality, they experience higher default rates.[15]

Another issue with a financial statement-based database is the possibility of a long lag between the last financial statement and the actual default event. Therefore, if a default is defined as an event within three to 15 months of the financial statement-based database, many defaults are omitted.

Figure 4 shows 34 percent of defaults occur more than 15 months after the last observation in the financial statement-based database. A calendar date-based database handles this issue more naturally. Figure 4 shows that only 7% of the default events occur more than 12 months after the last observation in the calendar date-based database. Consequently, the fact that the first default indicator is often the absence of a financial statement does not lead to a substantial downward bias in the default rate for a calendar date-based database, but it does for a financial statement-based database.

[15] Dwyer (2005) made this point as well.

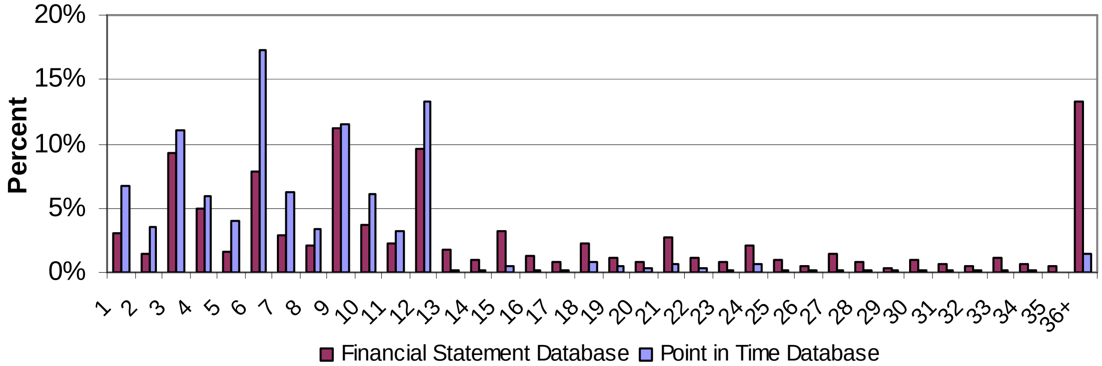

FIGURE 4 Months from Last Observation to Default

3.3 Data Filters

As with all of our analyses on private firm RiskCalc models, we exclude real estate, project finance, insurance, and financial firms. We also exclude not-for-profit organizations, government agencies, and firms with less than $100,000 in total assets. These firms fall outside the intended scope and are omitted from the rank order and level validation.

To provide a clean representation of the default characteristics, our data is extensively cleaned and verified. If the most recently available financial statement is more than two years old for any one obligor, we eliminate the observation.

Figure 5 displays the financial statement distribution by age. As seen in the figure, the percent of defaults and statements are similar for the 24+ group, which indicates that omitting this group does not significantly bias the default levels.

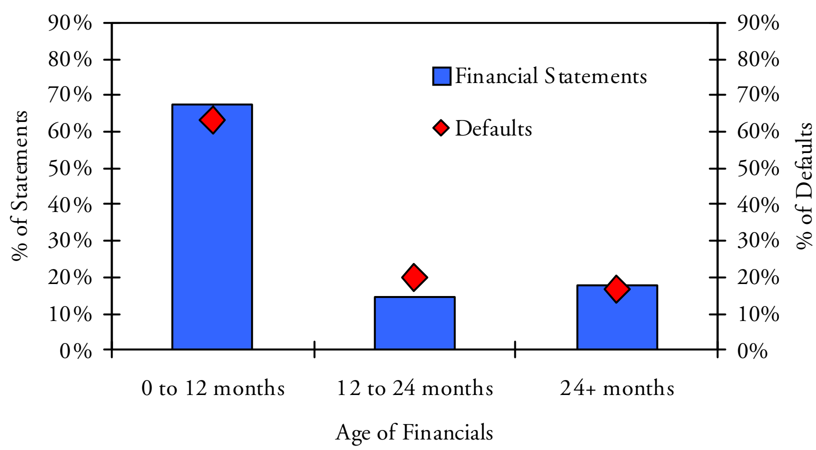

FIGURE 5 Financial Statement Distribution by Age

In addition, we exclude observations based on financial statements of low quality. Since we work with small- to medium-size private firms, a bank may accept a tax return, as well as draft or preliminary financial statements, as an indication of a firm's financial standing. We omit these records from the analysis due to their low quality. Table 2 summarizes the range of audit quality available and the categories included in our analysis.

TABLE 2 Financial Statement Quality

Quality (Best to Worst)	Statements
Unqualified Opinion	Included
Qualified Opinion	Included
Audited Statement	Included
Reviewed Statement	Included
Compiled Statement	Included
Company Prepared Statement	Included
Unaudited Statement	Included
Tax Return	Excluded
Drafts or Preliminary	Excluded

Figure 6 shows the financial statement distribution by complete statement history. For rank order validation, we further eliminate observations whose prior-year financial statement is unavailable. Line items from the prior statement are key inputs for several variables used in the RiskCalc v3.1 model. By eliminating these observations for rank order validation, the predictive power of the model increases somewhat and reflects the model power as if it were used with complete information. For purposes of level validation, we are reluctant to eliminate these observations because this omission could potentially bias the observed default rates (see Section 3.2).

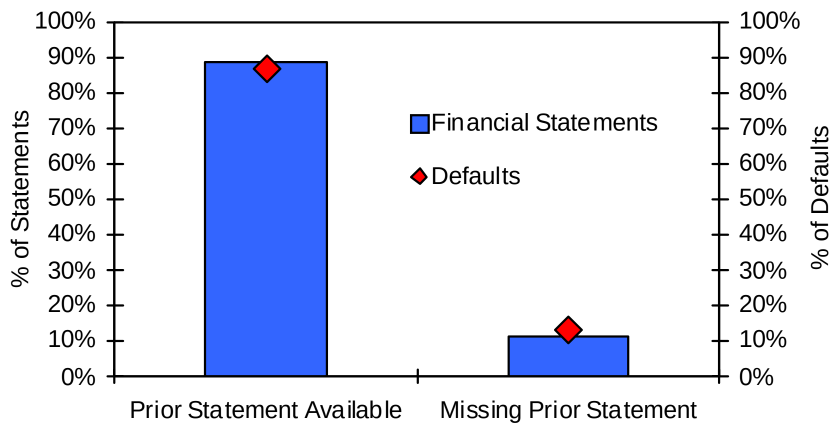

FIGURE 6 Financial Statement Distribution by Complete Statement History

3.4 Final Data Composition

Figure 7 presents financial statement and default distribution by year, and Table 3 provides the sample sizes. The increase in the number of observations and the default events over time represents improvements in the CRD coverage as well as increases in the extent of the loan accounting systems data that we capture from participating institutions.

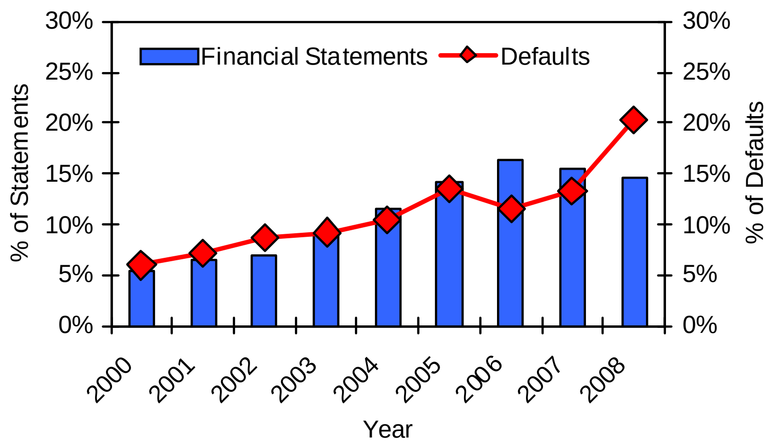

FIGURE 7 Financial Statement and Default Distribution by Year

TABLE 3 Information on Private Firm Sample Data

	RiskCalc v3.1 U.S. development dataset	RiskCalc v3.1 U.S. current validation dataset
Time period	1989-2002	2000-2008
Financial statements	183,000+	214,000+
Unique number of firms	40,000+	73,700+
Defaults	3,157	3,318

Figure 8, Figure 9, and Figure 10 present default and firm distribution by industry, size classification, and net sales, respectively. Firm size is measured by assets, which range from $100,000 to more than $50 million, and by net sales, which range from less than $500,000 to more than $50 million. These figures show that the proportion of defaults in one size group or industry is comparable to the number of firms in these groupings. Notable exceptions are Services and Construction; Services has proportionally lower defaults over the sample and Construction has proportionately more.

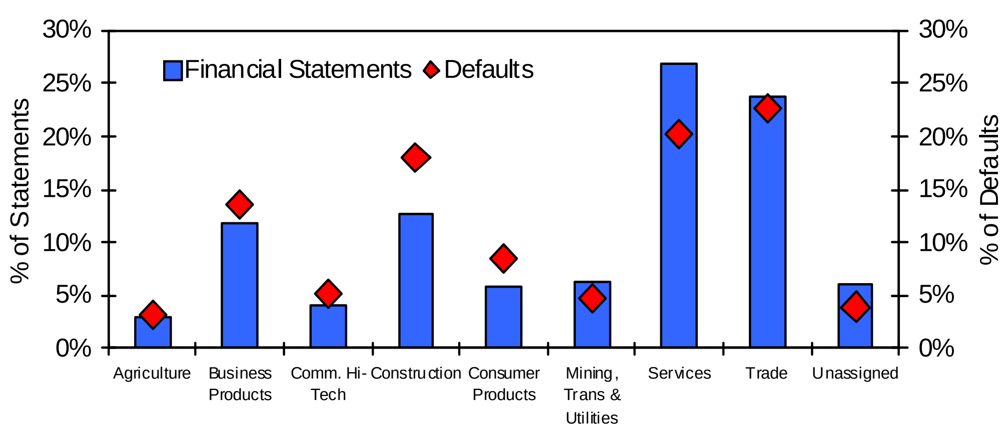

FIGURE 8 Default and Firm Distribution by Industry

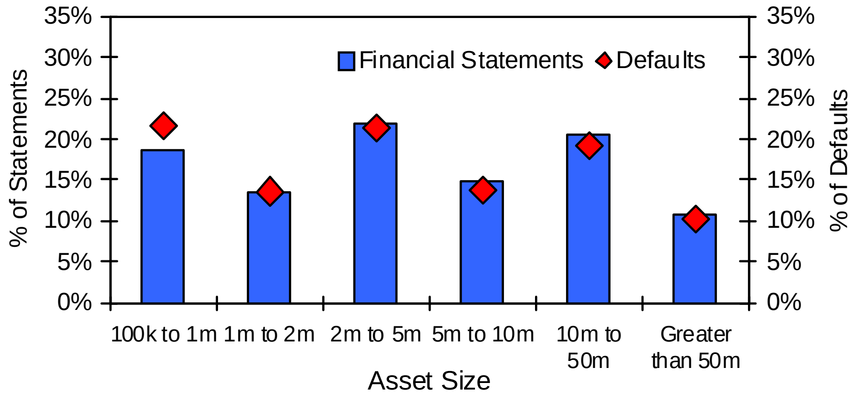

FIGURE 9 Default and Firm Distribution by Asset Size

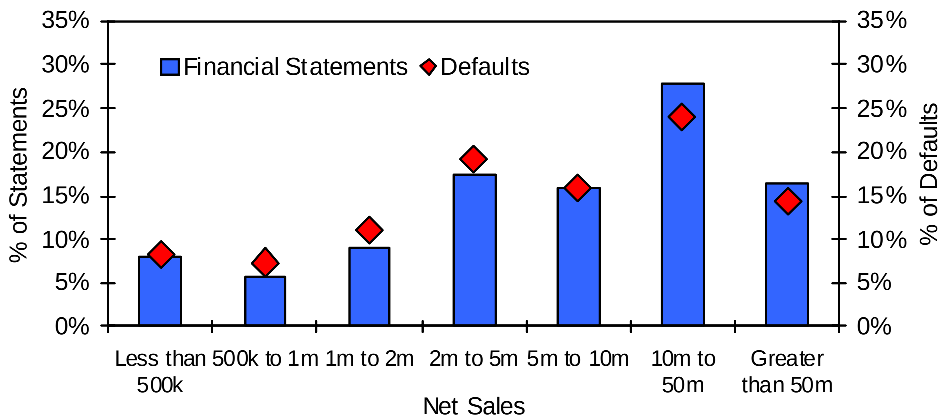

FIGURE 10 Default and Firm Distribution by Net Sales

4 RANK ORDER VALIDATION

In this section, we assess the model's ability to rank order credits using a Cumulative Accuracy Profile and the Accuracy Ratio (AR). We use Altman's Z-score as a reference model for comparison purposes.

Figure 11 presents the Cumulative Accuracy Profile s for the one-year model for the standard default definition and the extended definition, which includes defaults and near-defaults.[16] In this figure, the left-hand panel displays results over the whole sample (2000–2008). The right-hand panel presents results for 2003–2008, which is a pure out-of-sample performance test, as the model was developed in 2003.

[16] See Section 3.1 for definition of near-defaults.

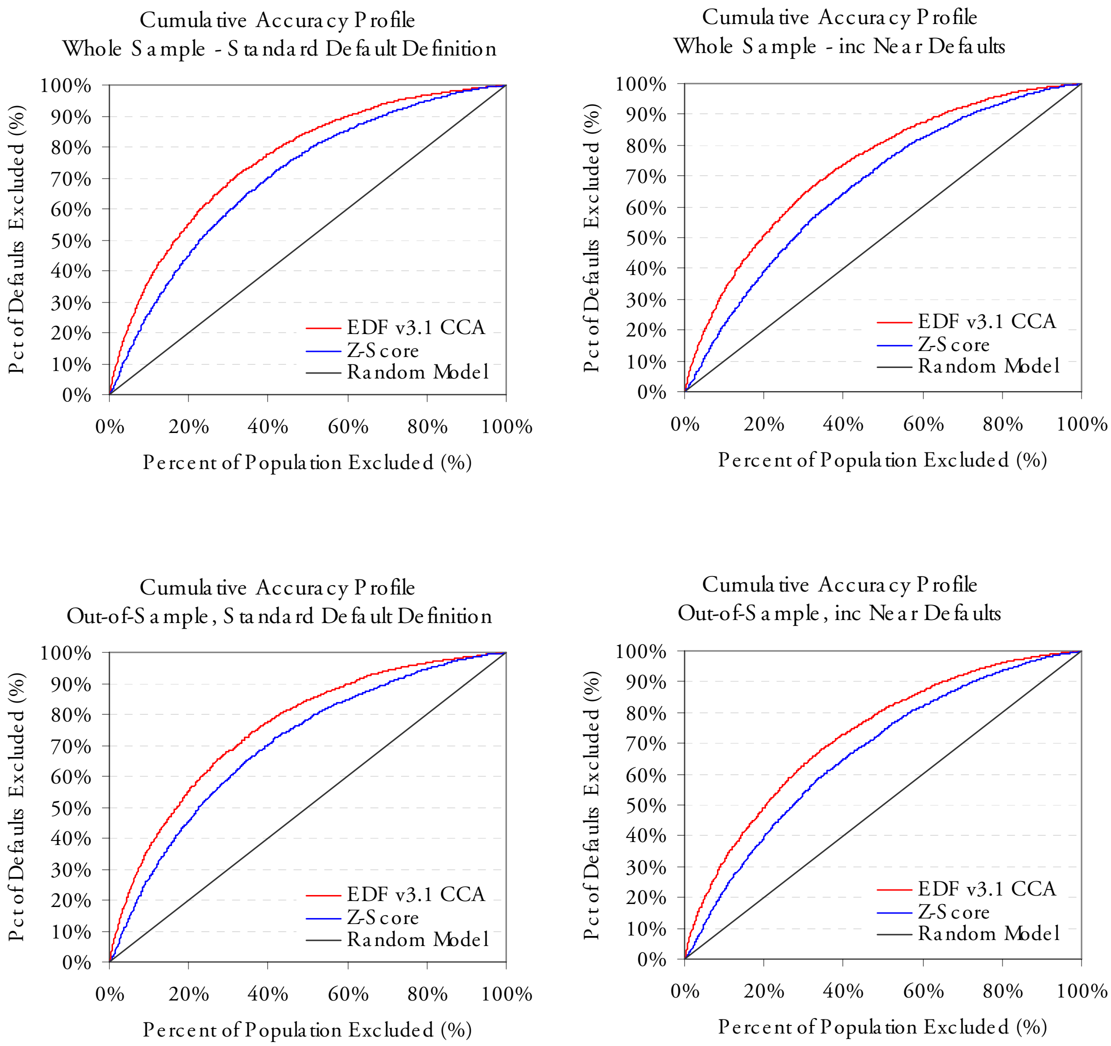

FIGURE 11 Cumulative Accuracy Profiles

Table 4 presents the corresponding accuracy ratios and sample statistics.

TABLE 4 Accuracy Ratios

	Full Sample (2000–2008)		**Out-of-Sample (2003–2008)**	
	Defaults	**Defaults & Near-defaults**	**Defaults**	**Defaults & Near-defaults**
RiskCalc v3.1 U.S.	52.0%	47.2%	51.7%	46.7%
Z-score	40.7%	34.0%	40.5%	34.1%
Defaults	3,318	5,180	2,451	3,615
Firms	73,700	71,800	63,700	61,500
Number of Statements	214,000	205,000	171,800	164,800

We see that the model provides significant discriminatory power and consistently outperforms Z-score, and that the performance has held up in a pure out-of-sample context. Both RiskCalc v3.1 U.S. and Z-score are more predictive of defaults than defaults and near-defaults by a significant margin (about five points). This difference is expected, as the first occurrence of a default event or a near-default event is often prior to the first occurrence of a default event. The later the default date in the bankruptcy process, the more likely it is that financial statements are indicative of financial distress. In some of our other RiskCalc country models, the only available default indicator is insolvency, and in such regions the power of the model is typically very high.[17]

Because models can be implemented at various points in the business cycle, we present power tests by year. These tests examine whether or not the model performance is excessively time dependent.

Table 5 displays the AR of RiskCalc v3.1 as compared to Z-score for each year at the one-year horizon for both definitions of default. RiskCalc v3.1 consistently outperforms Z-score by a considerable margin. The power of the model remains relatively stable over time.

TABLE 5 Model Power by Year

Defaults					
Year	Percentage of Defaults	RiskCalc v3.1	Z-Score	Difference in Models	p-value for Difference
2000	8.0%	53.8%	45.6%	8.2%	0.2%
2001	7.6%	55.0%	42.2%	12.8%	<.0001
2002	10.6%	48.9%	38.7%	10.2%	<.0001
2003	8.2%	59.2%	48.1%	11.1%	<.0001
2004	11.9%	58.7%	49.8%	8.9%	<.0001
2005	12.3%	60.7%	44.1%	16.6%	<.0001
2006	11.7%	48.4%	33.5%	14.9%	<.0001
2007	16.2%	43.2%	34.6%	8.6%	<.0001
2008	13.6%	51.3%	38.6%	12.7%	<.0001

Defaults and Near-defaults					
Year	Percentage of Defaults	RiskCalc v3.1	Z-Score	Difference in Models	p-value for Difference
2000	9.5%	46.6%	35.8%	10.8%	<.0001
2001	9.0%	45.8%	34.2%	11.6%	<.0001
2002	11.7%	43.2%	34.2%	9.1%	<.0001
2003	7.1%	56.6%	43.1%	13.4%	<.0001
2004	6.9%	48.6%	38.7%	9.9%	<.0001
2005	11.2%	49.7%	34.3%	15.4%	<.0001
2006	13.3%	46.1%	33.2%	13.0%	<.0001
2007	16.4%	39.7%	31.3%	8.4%	<.0001
2008	14.9%	44.4%	33.3%	11.1%	<.0001

It is important to test the model's overall power as well as its power among different industry segments and firm sizes.

Table 6 presents the power comparisons by sector for the one-year model. RiskCalc v3.1 outperforms Z-score in all sectors. The lowest power is in Agriculture while the highest is in Mining, Transportation, and Utilities.

[17] cf. Dwyer and Hood, 2004.

TABLE 6 Model Power by Industry

Defaults					
Industry	% of Defaults	RiskCalc v3.1	Z-Score	Difference in Models	p-value for Difference
Agriculture	3.0%	37.1%	32.6%	4.5%	27.6%
Business Products	13.8%	56.1%	49.6%	6.5%	0.0%
Communications and High Tech	5.2%	54.4%	45.8%	8.6%	0.4%
Construction	17.6%	53.8%	47.0%	6.8%	0.0%
Consumer Products	8.7%	53.8%	44.1%	9.6%	0.0%
Mining, Transportation, Utilities, and Natural Resources	4.8%	57.4%	49.4%	8.0%	1.3%
Services	19.5%	51.0%	40.1%	10.9%	<.0001
Trade	23.6%	48.9%	38.5%	10.4%	<.0001
Unassigned	3.7%	49.9%	30.7%	19.2%	<.0001

Defaults and Near-Defaults					
Industry	% of Defaults	RiskCalc v3.1	Z-Score	Difference in Models	p-value for Difference
Agriculture	3.7%	39.1%	30.8%	8.3%	2.0%
Business Products	15.1%	51.7%	40.4%	11.3%	<.0001
Communications and High Tech	4.3%	52.5%	41.7%	10.8%	<.0001
Construction	15.6%	47.5%	36.7%	10.8%	<.0001
Consumer Products	8.3%	49.5%	36.1%	13.4%	<.0001
Mining, Transportation, Utilities, and Natural Resources	5.3%	49.7%	39.2%	10.5%	<.0001
Services	19.5%	44.1%	34.7%	9.4%	<.0001
Trade	24.7%	45.1%	34.0%	11.1%	<.0001
Unassigned	3.5%	51.7%	30.6%	21.1%	<.0001

Table 7 and Table 8 present power comparisons by firm size (asset and net sales) for the one-year model. RiskCalc v3.1 out performs Z-score in all size groups. The highest power in the one-year model is found in the larger firms, with assets of more than $50 million; the lowest power is in the smallest firms, with assets or net sales under $500,000. This result is expected, as the quality of financial statements generally increases with firm size. For example, larger firms are more likely to have audited statements.

TABLE 7 Model Power by Size (Net Sales)

Defaults					
Size in Net Sales	**% of Defaults**	**RiskCalc v3.1**	**Z-Score**	**Difference in Models**	**p-Value for Difference**
<$500,000	6.2%	48.4%	30.6%	17.9%	<.0001
$500,000 to $1mm	6.8%	51.3%	37.8%	13.6%	<.0001
$1mm to $2mm	10.6%	49.4%	42.2%	7.2%	0.0%
$2mm to $5mm	19.0%	51.7%	40.9%	10.8%	<.0001
$5mm to $10mm	16.6%	54.8%	46.9%	7.9%	<.0001
$10mm to $50mm	26.4%	52.2%	42.8%	9.4%	<.0001
Over $50mm	14.4%	52.2%	32.3%	20.0%	<.0001

Defaults and Near-defaults					
Size in Net Sales	**% of Defaults**	**RiskCalc v3.1**	**Z-Score**	**Difference in Models**	**p-value for Difference**
<$500,000	5.7%	39.5%	21.0%	18.5%	<.0001
$500,000 to $1mm	5.4%	45.4%	33.1%	12.3%	<.0001
$1mm to $2mm	8.4%	49.8%	39.0%	10.8%	<.0001
$2mm to $5mm	18.0%	50.4%	37.2%	13.1%	<.0001
$5mm to $10mm	16.8%	51.5%	39.3%	12.2%	<.0001
$10mm to $50mm	29.6%	47.9%	36.8%	11.2%	<.0001
Over $50mm	16.1%	46.4%	26.8%	19.6%	<.0001

TABLE 8 Model Power by Size (Total Assets)

Defaults					
Size of Assets	**% of Defaults**	**RiskCalc v3.1**	**Z-Score**	**Difference in Models**	**p-value for Difference**
$100,000 to $1mm	19.7%	50.3%	38.7%	11.6%	<.0001
$1mm to $2mm	13.6%	54.0%	46.3%	7.7%	<.0001
$2mm to $5mm	22.2%	51.2%	41.9%	9.3%	<.0001
$5mm to $10mm	14.3%	52.2%	40.1%	12.1%	<.0001
$10mm to $50mm	20.4%	51.5%	39.7%	11.8%	<.0001
Over $50mm	9.9%	56.0%	36.3%	19.7%	<.0001

Defaults and Near-defaults					
Size of Assets	**% of Defaults**	**RiskCalc v3.1**	**Z-Score**	**Difference in Models**	**p-value for Difference**
$100,000 to $1mm	16.1%	44.7%	33.4%	11.3%	<.0001
$1mm to $2mm	12.9%	49.0%	39.5%	9.5%	<.0001
$2mm to $5mm	23.0%	50.7%	37.9%	12.8%	<.0001
$5mm to $10mm	16.2%	49.0%	35.2%	13.8%	<.0001
$10mm to $50mm	22.1%	45.6%	30.8%	14.8%	<.0001
Over $50mm	9.8%	49.0%	29.2%	19.8%	<.0001

Table 9 highlights the accuracy ratios for the standard definition of default by financial statement age. As shown in this chart, the model maintains its power when relying on statements as old as two years; after this period, the discriminatory power of the model decreases. This result is also expected, as the model's financial inputs are less representative of the current financial situation.

TABLE 9 Model Power by Financial Statement Age

Group	Total Defaults	RiskCalc v3.1
Expanded Sample[18]	100%	47.7%
0m-12m	62.6%	53.4%
12m-24m	20.4%	46.5%
24m-36m	6.6%	35.1%
36m+	10.4%	20.7%

5 LEVEL VALIDATION

Over the centuries, the definition of a probability has been influenced by the objective of determining the odds of specific games of chance, such as dice games, which have interested scientists including Galileo, Leibniz, Fermat, and Pascal. For example, if a gambler asks a statistician the odds of snake eyes, the statistician will point out that if the dice are fair and independent, then the odds of snake eyes are 1-in-36.[19] Further, the statistician can tell the gambler if the dice are rolled 3600 times, the odds are 19-in-20 a snake eyes will result between 80 and 120 times.[20] Finally, if a statistician observed an outcome outside of this interval, she would reject the null hypothesis that dice are fair and independent, and recognize that by doing so there is a 5% chance of a Type 1 error—rejecting the null when it is in fact true.

Defaults are more complicated than dice, however, as events tend to happen in clusters with interdependent dynamics. Even if a model is correct, you may frequently observe more than 120 defaults, or less than 80 defaults in a portfolio of 3600 exposures with a 1-in-36 chance of default in a given year. The level validation of a PD model is assessing whether or not the observed default rate is consistent with the level of the PD produced by the model.

A number of challenges develop when defining a default rate. Numerous approaches exist, and each approach leads to different default rates. As shown in Section 3.2, a default rate constructed using a financial statement based database will be lower than a calendar date-based database, unless additional assumptions are made regarding the handling of missing financial statements, as well as financial statements for firms not actively borrowing. We can directly address these issues using a calendar date-based database that uses loan accounting information to determine if a firm was a borrower from the bank. Therefore, in this paper, we compute the default rate using the calendar date-based database. The denominator is the population of firms actively borrowing with a RiskCalc EDF credit measure based on a financial statement no more than two years old as of April 1 of the given year. The numerator is the number of these firms that we record as defaulting over the next 12 months.

Some issues remain when using a calendar date-based approach:

1. We do not capture a defaulting firm if it defaults with a different creditor.
2. We do not capture instances when a borrower pays for a loan at one bank using a loan from another bank, and then defaults on the second loan.

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[18] For a comparison of accuracy ratios by age of financial statements, the sample included statements older than 24 months. The prior analyses in this section did not include these older financials statements.

[19] "Snake eyes" is a gambling term for a {1,1} outcome when rolling two six-sided dice.

[20] This approximation uses the central limit theorem.

3. We use only borrowers that we have successfully matched with the loan accounting system in the cohort, creating a possible sample selection bias.[21]

4. It can be challenging to distinguish between a stock of classified borrowers and a flow of firms defaulting during the year in a loan accounting system. These issues can arise when loan accounting systems are integrated following an acquisition, which may add a large stock of classified and non-classified borrowers. We minimize this issue by excluding defaults if the default date cannot be verified. Nevertheless, it is difficult to eliminate such issues.

Despite these issues, in each year, we track a specific cohort of borrowers, monitor whether they are current on their debts, and then define a default rate accordingly. Therefore, banks would use a default rate similar to that which we calculate for risk management purposes.

After computing the realized default rate, we use two approaches for level validation. The first approach estimates the correlation parameter implied by the realized default rates and the model PD values. If the observed default rate can only be rationalized with a large correlation parameter, this may suggest that we should revisit the calibration.

The second approach estimates a PD adjustment factor and a correlation parameter given the realized default rates and the model PD values. To the extent the PD adjustment factor is close to one, the level of the PD is consistent with the realized default rates. If the PD adjustment factor is less than one, this is evidence that the model is conservative relative to the default rate observed on this sample. If the PD adjustment factor is greater than one, this suggests the PD values produced by the model may, in fact, be too low. We evaluate both our standard definition of default and the expanded definition, which includes near defaults. We use a single-factor Gaussian model for our correlation framework (See Appendix A).

The observed default rates are for all active borrowers with defaults from January 2000–December 2008. To account for an incomplete default window for the 2008 year (i.e., missing January 1, 2009–March 31, 2009) we have multiplied the aggregate default rate by 4/3.

5.1 Approach 1: Estimating a Correlation Parameter

Figure 12 presents the time series of actual default rates against the distribution of default rates implied by the model, with correlation assumed at 0% and 10%. The 10th percentile, the median, the average, and the 90th percentile represent the distribution of the default rate. With a zero correlation parameter, the expected default rate distribution for each year is narrow, and the model predicts a very specific range of default rates. As we increase the correlation parameter, the distribution of defaults predicted by the model becomes more dispersed. This displays the relationship between the correlation parameter and the variability of the expected default rate.

[21] For the borrowers in the loan accounting system data, we can find financial statements for approximately 20%. For the financial statements in our database, we can find corresponding borrowers for more than 50%. The reasons we cannot match financial statements to borrowers include: (i) some banks may not require that the financial statements of smaller firms are "spread" into their system, (ii) banks are improving their process for matching financial statement data base information to their loan accounting system information, (iii) financial statements may be entered into the system as part of a prospective borrower applying for a loan, but in fact the loan was never made.

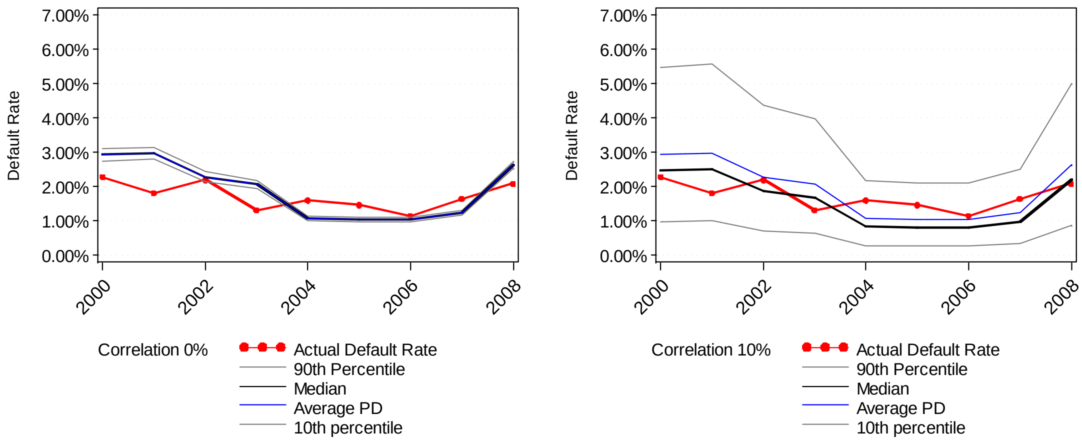

FIGURE 12 Historical Default Rate and the Distribution of Predicted Default Rates for Different Correlation Assumptions

The left-hand panel in Figure 12 shows that, with an asset correlation of zero, the realized default rate falls outside the model's predicted 10th and 90th percentile in eight out of nine years. The odds of a one-in-five event occurring in eight or more times out of nine are 1.9 x 10-5.[22] This low probability is evidence of correlation; if defaults were uncorrelated and the model were correct, we would expect the average PD value to track the realized default rates much more closely.

The right-hand panel in Figure 12 shows that by assuming a 10% asset correlation, in all observed years the actual default rate is within the 10th and 90th percentile of the range of defaults predicted by the model. If both the model's EDF credit measures and the correlation assumption are correct, the actual default rate should fall outside the 10th and 90th percentiles on average one out of every five years. The odds of the default rate not falling outside this range in nine out of nine years are 0.89, approximately 13%. This finding implies that the correlation assumption may be too large.

We employ a maximum likelihood method to estimate the correlation parameter, consistent with the observed default series.[23] Figure 13 graphs the expected distribution using the fitted correlation for the two definitions of default. The left-hand panel is based on defaults and on a correlation parameter of 2%, and shows the standard definition of default. The right-hand panel includes both defaults and near-defaults and is based on a correlation parameter of 10%.

[22] This probability is determined by the binomial distribution, where the probability of being within the 10th and 90th percentile is 80%, and each year is assumed independent: $\binom{9}{0}0.2^9 \times 0.8^0 + \binom{9}{1}0.2^8 \times 0.8^1$.

[23] For details, see Appendix A.

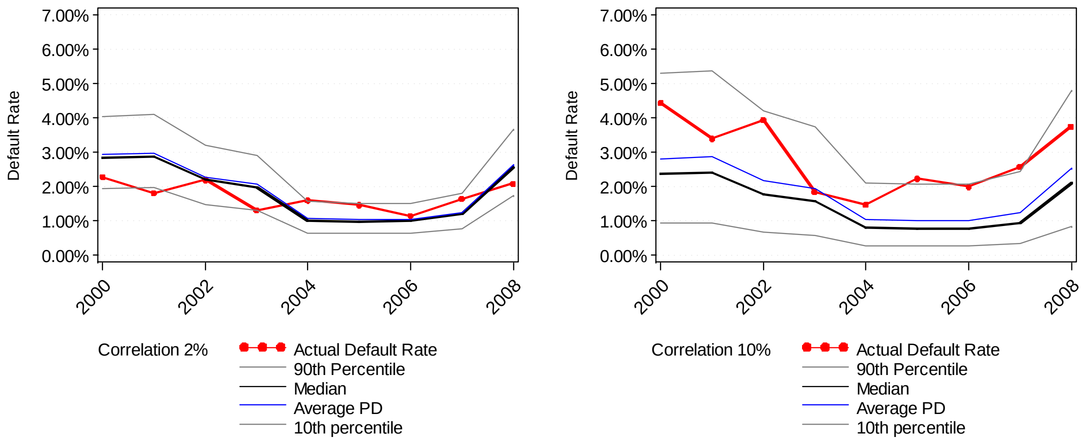

FIGURE 13 Distribution of Expected Default Rates with Fitted Correlation vs. Actual Default Rates

Using an asset correlation of 2%, the realized default rate is most consistent with the modeled default rates over the last nine years, when using the standard definition of default. With this asset correlation, the realized default rate fluctuates around the median predicted default rate, with appropriate variability. In our view, an asset correlation of 2% is reasonably low. Consequently, the PD level is not inconsistent with the observed default rate on this population.

When including both defaults and near-defaults (shown in the right-hand panel in Figure 13), we estimate an asset correlation of 10%. An asset correlation of 10% falls within the range of what most people use in the context of a portfolio model. However, the realized default rates are consistently above the median default rate implied by the model. This level indicates that the PD values produced by the model are low when compared to observed default rates that include both defaults and near-defaults. In Section 5.2, we extend our correlation fitting method to allow for a multiplicative adjustment to the PD and simultaneously solve for the correlation and adjustment factor most consistent with the realized default rates. The average of the time series of average EDF credit measures on this sample is 1.8%. This average compares with an average default rate of 1.6% (the average of the red line on the left-panel in Figure 12) and an average "default and near-default" rate of 2.8% (the average of the red line on the right-panel in Figure 12.

A notable deviation from the median is 2003—a year of economic recovery that led to an unexpectedly low number of defaults. 2003 includes all active borrowers in 2003 and defaults from April 1, 2003–March 31, 2004. To explain the low default rate, we use methods discussed in Dwyer (2007) to determine the implied posterior distribution of the aggregate shock to the economy. We estimate that the U.S. economy received a positive 1.3 standard deviation shock relative to market expectations. This finding is consistent with the stock market recovery observed after the recession following the tech bubble crash.

Figure 14 displays this posterior distribution for the aggregate shock given the actual default rate and p-values of the actual default rate, which is the probability of observing a default rate at or lower than the actual default rate. P-values are between 10% and 80%.

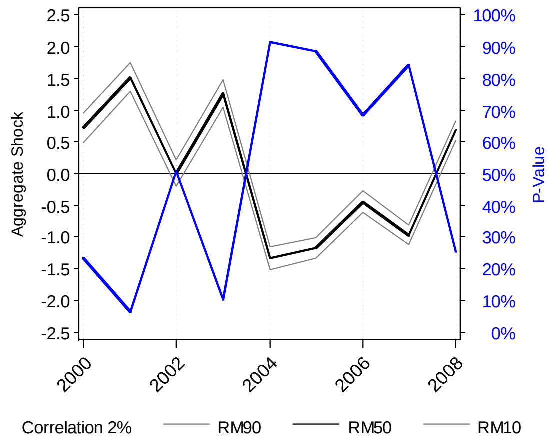

FIGURE 14 Posterior Distribution of the Aggregate Shock

5.2 Approach 2: Estimating both Correlation and a PD Adjustment

To further test the PD level, we can simultaneously solve for a multiplicative adjustment factor and a correlation factor that maximizes the log-likelihood. The problem is now a two-dimensional maximization problem amenable to a grid search, as the PD factor is close to one and the correlation parameter is bound between zero and one.

Figure 15 presents the surface of the log-likelihood function across a PD adjustment factor and a correlation parameter. The surface is well-behaved and obtains a clear maximum.

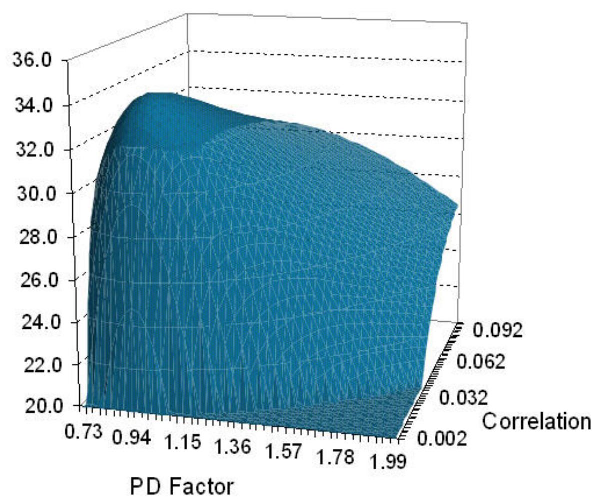

FIGURE 15 Maximization Surface

Figure 16 presents the time series of realized versus predicted defaults after estimating both a PD adjustment factor and a correlation parameter. The left-hand panel shows the results when using the standard definition of default. For this sample, the maximum likelihood method implies a PD factor of 1.0, and an optimal correlation parameter is 2%, indicating the PD level is consistent with the realized series. The right-hand panel shows results when including both defaults and near-defaults. For this extended default definition, the PD is too low; multiplying the PD by 1.5 yields the best fit to the realized series. If we make this adjustment, then the estimated correlation parameter becomes reasonably low as well (0.02).

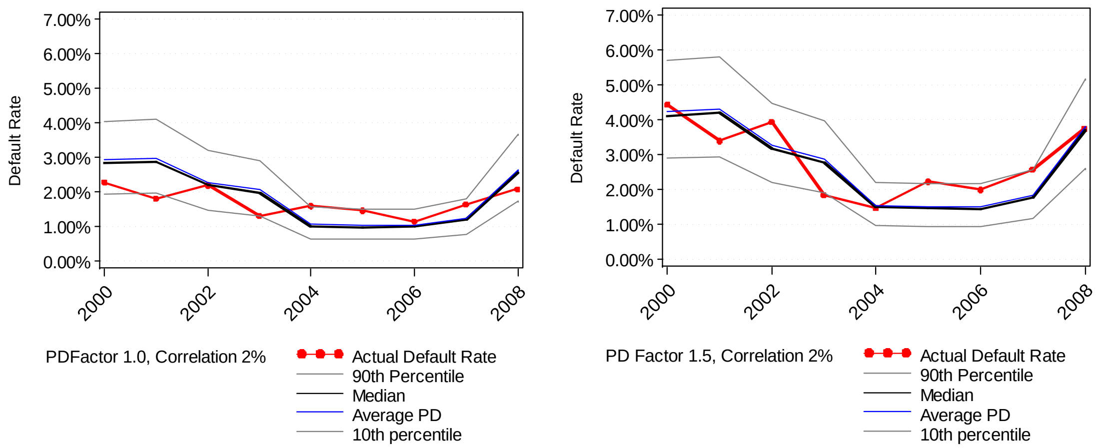

FIGURE 16 Expected Default Rates with Fitted PD Factor and Correlation vs. Actual Default Rates

6 CONCLUSION

From a rank ordering standpoint, the performance of the RiskCalc v3.1 United States model continues to be robust. The rank ordering results are robust when default is defined as both default and default plus near-defaults. In addition, the model outperforms Altman's Z-score along every segment. Not surprisingly, the accuracy ratios are higher when predicting defaults as opposed to defaults and near-defaults.

The level of the default probability is consistent with the realized default rates after allowing for a reasonably small amount of systematic risk. We can capture such systematic risk using a single-factor model, and the amount of systematic risk required is reasonably low.

It is important to note that we validate the model using data from actual lenders who decide to originate a loan. In most cases, these lenders hold the loans on their balance sheets. Further, prior to making the loan, they consider the loan application as a whole. Presumably, the firms to which the CRD participants chose to lend are of higher quality than firms whose applications are declined. We are not aware of any bank in the CRD that lends based solely on the borrower's RiskCalc EDF credit measure. If we base the loan origination decision solely on the RiskCalc EDF credit measure without additional due diligence, the model might perform differently.

This paper shows the value of using current financial statements to monitor credits. Active borrowers with old financial statements are more likely to default. In addition, the ability of a lender to rank order their borrowers in terms of credit risk declines as the financial statements age. There appears to be a return to monitoring the age of the financial statements used to evaluate the risk of borrowers and asking the borrowers for current financial statements as appropriate.

Finally, this paper validates the model on the population the model was intended for: small- to medium-sized enterprises. Please refer to Dwyer and Zhao (2009) for information about how this model works when applied to very large firms.

APPENDIX A LIKELIHOOD ESTIMATION OF A CORRELATION PARAMETER

The standard Gaussian single factor framework can be viewed as decomposing changes in distance-to-default for a firm into a function of a single factor that models the systemic risk, with a residual representing the idiosyncratic risk of an individual exposure.[24] Suppose that each firm's PD can take on one of the values in the set $\{PD_1, PD_2, \ldots, PD_N\}$. Let the correlation parameter be denoted by ρ. The probability of default given the aggregate shock for PD_i is given by:

$$PD_{i|\phi} = N\left(\frac{N^{-1}(PD_i) - \sqrt{\rho}\phi}{\sqrt{1-\rho}}\right) \quad (1)$$

In the case of homogeneous PD values, the distribution of defaults given the random shock is a binomial distribution and is straightforward to work with. The exact default distribution, given the aggregate shock in the case of heterogeneous PD values, is more involved.[25] If the sample size is sufficiently large and the PD values are sufficiently big, we can invoke a law of large numbers approximation. A portfolio's default rate distribution, given the aggregate shock, ϕ, can be approximated by a normal distribution with a mean and variance given by:

$$\mu_\phi = \sum_{i=1}^{N} w_i PD_{i|\phi}; \quad \sigma_\phi^2 = \sum_{i=1}^{N} w_i^{\,2} Var_{i|\phi} \quad (2)$$

Where w_i is the proportion of the exposures in PD bucket i, $Var_{i|\phi} = PD_{i|\phi}(1 - PD_{i|\phi})/N_i$ and N_i is the number of exposures in PD bucket i. Under this approximation, the probability density function of the distribution of defaults given the random shock is given by:

$$\frac{n\left(\left(s - \mu_\phi\right)/\sigma_\phi\right)}{\sigma_\phi} \quad (3)$$

Where s is the sample default rate and n is the density function for a standard normal distribution. Given these approximations, the unconditional density function of the sample default rate is determined by "integrating out" the aggregate shock:

$$f(s;\phi) = \int_{\phi=-\infty}^{\infty} \frac{n\left(\left(s - \mu_\phi\right)/\sigma_\phi\right)}{\sigma_\phi} n(\phi) d\phi \quad (4)$$

The above formulas can be calculated easily using numerical integration. For example, the 2002 probability density would appear as shown in Figure 17 for a 4% asset correlation.

---

[24] One of the early applications of this framework was for modeling the distribution of the losses in a credit portfolio. Oldrich Vasicek solved the limiting distribution in a note that is dated 1987. This note was later extended and published in *Risk* in 2002. This distribution provides the foundation of the regulatory capital formula used in the Basel Capital Accord (Basel Committee, 2005).

[25] cf. Hull and White, 2004.

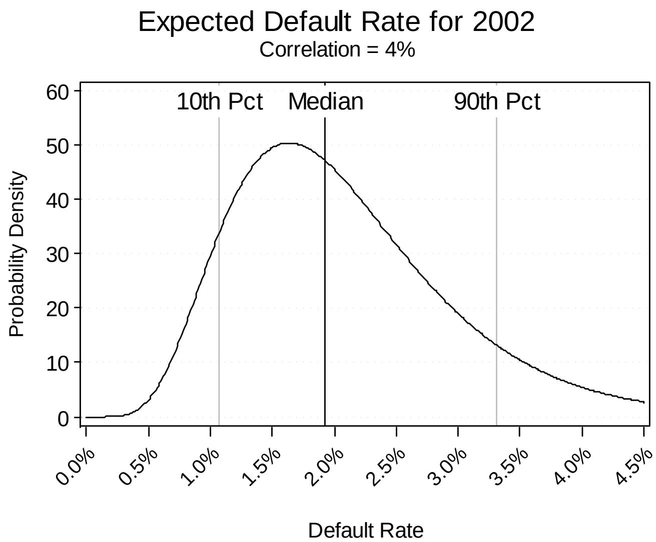

FIGURE 17 Expected Default Rate for 2002

The realized default rate for 2002 was 1.9%, which, according to this distribution, implies a probability density of 47.1%. Given a time series of sample default rates and the correlation assumption $(s_1, \ldots, s_T; \rho)$, we can easily calculate the log likelihood of this time series as the sum of the logarithm of each probability density:

$$l(s_1, \ldots, s_T; \rho) = \sum_{i=1}^{T} \log(f(s; \rho)) \quad [5]$$

We can then use numerical techniques to solve for the ρ that is most likely given the observed sample default rates and the model's PD values. If this correlation turns out to be very large, this is likely to be indicative of issues with the PD model.

We can extend this method to allow for a scaling factor on the probability of default. We can adjust the PD values by a multiplicative adjustment factor, and then simultaneously solve for the adjustment factor and the correlation coefficient that maximize the likelihood function.

Under this extension, the probability of default given the aggregate shock becomes:

$$PD_{i|\phi} = N\left(\frac{N^{-1}(\varphi \times PD_i) - \sqrt{\rho}\phi}{\sqrt{1-\rho}}\right) \quad [6]$$

In addition, modifying the mean and variance of the sample default rate and the unconditional default density accordingly, the likelihood function becomes:

$$l(s_1, \ldots, s_T; \rho, \varphi) = \sum_{i=1}^{T} \log(f(s; \rho, \varphi)) \quad [7]$$

APPENDIX B BASEL DEFAULT DEFINITION

In this section, we reprint the Basel definition and the definition of default written by the U.S. regulatory agencies in *Risk-Based Capital Standards: Advanced Capital Adequacy Framework—Basel II; Final Rule*.

Basel Definition of Default

Paragraphs 452–454 of "A Revised Framework" state:

> 452. A default is considered to have occurred with regard to a particular obligor when either or both of the two following events have taken place.
>
> - The bank considers that the obligor is unlikely to pay its credit obligations to the banking group in full, without recourse by the bank to actions such as realizing security (if held).
> - The obligor is past due more than 90 days on any material credit obligation to the banking group. Overdrafts will be considered as being past due once the customer has breached an advised limit or been advised of a limit smaller than current outstandings.
>
> 453. The elements to be taken as indications of unlikeliness to pay include:
>
> - The bank puts the credit obligation on non-accrued status.
> - The bank makes a charge-off or account-specific provision resulting from a significant perceived decline in credit quality subsequent to the bank taking on the exposure.
> - The bank sells the credit obligation at a material credit-related economic loss.
> - The bank consents to a distressed restructuring of the credit obligation where this is likely to result in a diminished financial obligation caused by the material forgiveness, or postponement, of principal, interest or (where relevant) fees.
> - The bank has filed for the obligor's bankruptcy or a similar order in respect of the obligor's credit obligation to the banking group.
> - The obligor has sought or has been placed in bankruptcy or similar protection where this would avoid or delay repayment of the credit obligation to the banking group.
>
> 454. National supervisors will provide appropriate guidance as to how these elements must be implemented and monitored. (Basel Committee on Banking Supervision, "International Convergence of Capital Measurement and Capital Standards ['A Revised Framework']," Bank for International Settlements, 2004).

U.S. Regulatory Agencies Definition of Default

The regulatory bodies in the U.S. use a very similar language in *Risk-Based Capital Standards: Advanced Capital Adequacy Framework—Basel II; Final Rule* that they state is intended to be consistent with the "New Accord." Page 69307 of the December 7, 2007 Federal Registry states:

> The bank considers that the obligor is unlikely to pay its credit obligations to the bank in full, without recourse by the bank to actions such as realizing collateral (if held); or (ii) the obligor is past due more than 90 days on any material credit obligation to the bank. The final rule also clarifies, consistent with the New Accord, that an overdraft is past due once the obligor has breached an advised limit or has been advised of a limit smaller than the current outstanding balance.
>
> Consistent with the New Accord, the following elements may be indications of unlikeliness to pay under this definition:
>
> (i) The bank places the exposure on non-accrual status consistent with the Call Report Instructions or the TFR and the TFR Instruction Manual;

(ii) The bank takes a full or partial charge-off or write-down on the exposure due to the distressed financial condition of the obligor;

(iii) The bank incurs a material credit-related loss in connection with the sale of the exposure or the transfer of the exposure to the held-for-sale, available-for-sale, trading account, or other reporting category;

(iv) The bank consents to a distressed restructuring of the exposure that is likely to result in a diminished financial obligation caused by the material forgiveness or postponement of principal, interest or (where relevant) fees;

(v) The bank has filed as a creditor of the obligor for purposes of the obligor's bankruptcy under the U.S. Bankruptcy Code (or a similar proceeding in a foreign jurisdiction regarding the obligor's credit obligation to the bank); or

(vi) The obligor has sought or has been placed in bankruptcy or similar protection that would avoid or delay repayment of the exposure to the bank. (Office of the Comptroller of the Currency [OCC], the Board of Governors of the Federal Reserve System [Board], the Federal Deposit Insurance Corporation [FDIC], and the Office of Thrift Supervision [OTS]. "Risk-Based Capital Standards: Advanced Capital Adequacy Framework—Basel II; Final Rule," Federal Registry, vol. 72, no. 235, December 7, 2007).

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