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MOODY'S KMV RISKCALC™ V3.1 U.S. BANKS

MODELING**METHODOLOGY**

ABSTRACT

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Moody's KMV RiskCalcTM is the Moody's KMV model for predicting private company defaults. It covers over 80% of the world's GDP, has more than 20 geographic specific models, and is used by hundreds of institutions worldwide. While using the same underlying framework, each model reflects the domestic lending, regulator, and accounting practices of its specific region.

This document outlines the underlying research, model characteristics, data, and validation results for the latest addition to the RiskCalc suite of models, RiskCalc v3.1 U.S. Banks model. The model assesses the risk of banks, savings and loans, and thrifts as well as bank holding companies. Currently, there are more than 13,000 legal entities covered. The assets of the largest banks exceed $1 trillion while the smallest are less than $15 million.

The U.S. Banks v3.1 model incorporates a larger database over a longer time period than its predicessor. The ratios and credit cycle adjustment have been refined. The expanded validation of the model includes smaller banks that the U.S. Banks v1.0 model did not handle. Subsidiaries of publicly traded banks can be evaluated as standalone entities, while at the same time the model provides a view on the credit risk of the publicly traded parent that is based on the Moody's KMV Financial Firm Model.

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1 INTRODUCTION

The original RiskCalc v1.0 U.S. Banks model was released in early 2002 and was based on data available through the end of 2000 (Kocagil et al., 2002). In the four years since the release, the model has been actively used in the process of rating trust preferred security (TruPS) backed collateralized debt obligations.[1] The v1.0 model was designed to work for stand-alone private banks, public banks, and hybrids, which are private banks that are owned by publicly traded parents. The U.S. Banks v1.0 model was the first RiskCalc model to include a credit cycle adjustment factor.

The U.S. Banks v3.1 model is built on a comprehensive database and utilizes Moody's KMV extensive research, including a credit-cycle adjustment that is based on the Moody's KMV public firm financial model. The database combines banking and thrift data from the Federal Deposit Insurance Corporation (FDIC), and bank holding company data from the Federal Reserve Bank. In the four years since the U.S. Banks v1.0 model was released, we have been able to expand the scope of the model and extend the transparency of our modeling framework (see Dwyer, Kocagil and Stein, 2004; and Dwyer and Stein, 2005).

RiskCalc Modes

RiskCalc v3.1 allows a user to assess the risk of a privately-held bank in two ways: Financial Statement Only (FSO) and Credit Cycle Adjusted (CCA).

The FSO mode delivers a bank's default risk based only on regulatory filings. In this mode, the risk assessments produced by the model are relatively stable over time.

The CCA mode adjusts the default risk by taking into account the current stage of the credit cycle. The CCA adjustment is a sector-specific factor derived directly from the Moody's KMV Financial Firm model's distance-to-default. The CCA model reflects the market's current assessment of the credit cycle and is a forward-looking indicator of default.

The CCA adjustment is specific to the firm's sector and country and is updated monthly. The CCA mode also has the ability to stress test EDF credit measures under different credit cycle scenarios – a requirement under Basel II.

U.S. Banks v3.1 Model versus U.S. Banks v1.0 Model

Since the release of the U.S. Banks v1.0 model, the size of the database for U.S. privately-held banks has substantially increased. The new model refines both the financial statement variables and the credit cycle adjustment that were in the original model. We also made substantial advances in our model development and testing techniques. As a result, the new model is more powerful and precise than its predecessor. It also includes additional analytic tools that increase model usability and transparency.

The original model used a first generation public firm model to evaluate the risk of public banks, and used a combination of this model and the private banking model to evaluate the risk of hybrid banks. The new model evaluates the risk of public banks using the Moody's KMV Financial Firm Model. For hybrid banks, it provides an evaluation of the risk of a private bank as though it were a standalone entity. Further, it provides a pass through to the parent bank and its corresponding EDF. This allows the user to decide how much weight to place on the credit risk of the private bank versus the credit risk of the publicly traded parent. We anticipate that the user will want to choose this weight on the basis of the legal entity to which they would have legal recourse in the event of default.

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[1] Brennan and Leahy (2006) provide a recent description of the development of the U.S. Trust Preferred CDO sector over the past seven years.

2 DATA DESCRIPTION

The U.S. Banks v3.1 model is constructed on information collected from multiple data resources including FDIC's Research Information System (RIS) database and the Federal Reserve Bank's bank data. The RIS database covers all FDIC-insured legal entities (e.g., a bank or a thrift).[2] Federal Reserve data covers bank holding companies.[3]

The structure of the data available for this model is heavily influenced by how the banking industry is regulated and the reports that institutions are required to file. The banks insured by the FDIC through Bank Insurance Fund (BIF) are regulated by one of the following agencies: the Office of the Comptroller of Currency (OCC), the Federal Reserve Bank and the FDIC. Banks are required by the Federal Financial Institution Examination Council (FFIEC) to file consolidated Reports of Condition and Income (Call Report) on a quarterly basis. The thrift institutions insured by FDIC through Savings Association Insurance Fund (SAIF) are regulated by the Office of Thrift Supervision. They are required to file a consolidated Thrift Financial Report (TFR) on a quarterly basis. The Federal Reserve has responsibility for the oversight of bank holding companies under the Bank Company Holding Act. Bank holding companies are required to file financial reports (Y9 series) on a quarterly basis.[4]

2.1 Definition of Default

RiskCalc uses FDIC resolutions (also known as bank failure & assistance) to determine bank defaults.[5,6] Based upon the resolution method employed to protect insured depositors of assisted banks and how each transaction affects an assisted institution's charter, the default types outlined below can be grouped into three general categories.

- *Assistance Transactions*. In most assistance transactions, insured and uninsured depositors are protected, the assisted institution remains open, and its charter survives the resolution process.
- *Purchase and Assumption Transactions*. In purchase and assumption transactions, the failed institution's insured deposits are transferred to a successor institution, and its charter is closed. In most of these transactions, additional liabilities and assets are also transferred to the successor institution.
- *Payoff Transactions*. In payoff transactions, the deposit insurer—the FDIC or the former Federal Savings and Loan Insurance Corporation—pays insured depositors, the failed institution's charter is closed, and there is no successor institution.

In our bank failure database, records of FDIC resolutions are only associated with the entities insured by the FDIC. To validate the use of the model on bank holding companies, which are not insured by FDIC, we constructed the following definition of default. We defined a holding company as being in default if a large portion of its subsidiary banks receive FDIC assistance in a short-time period. Specifically, for each holding company, we began with pooling assistance records of its subsidiary banks. We defined the holding company as being in default if one-half of its subsidiaries defaulted

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[2] Includes savings banks and savings and loan associations.

[3] A Bank Holding Company (BHC) is defined under Bank Holding Company Act of 1956 as a company that either owns outright or has a controlling interest in a bank. In 2004, there were approximately 6,000 bank holding companies.

[4] Companies that own FDIC-insured entities are not necessarily covered in our bank and bank holding company database. One example is commercial companies that control industrial loan companies and industrial banks (collectively, ILCs) that are FDIC-supervised. Examples include GMAC Automotive Bank or Target Bank. Their ultimate parents, General Motors Corporation and Target are neither Bank Holding Companies nor Financial Holding Companies.

[5] In the United States, a bank or thrift institution must obtain a charter from a recognized chartering authority in order to obtain federal deposit insurance and do business. The chartering authority typically closes an institution when the institution becomes insolvent, critically undercapitalized, or unable to meet requests for deposit withdrawals. The chartering authority informs the FDIC when an insured institution will be closed and the formal FDIC resolution activities begin.

[6] This definition of default differs from typical definition of a corporate default. Prior to the FDIC Improvement Act of 1991 (FDICIA), the government largely protected uninsured deposits at banks upon failure. Following the FDICIA, the government has not always protected uninsured deposits. Consequently, recent bank failures are often defaults in the sense that large depositors actually lost money. As of the beginning of 1995, in fact, the FDIC is prohibited from protecting uninsured depositors or creditors if doing so would increase the cost to the fund. Under certain circumstances, the FDIC can invoke a *too-big-to-fail* exemption from this rule (cf., Bentson and Kaufman 1997). For purposes of this document, we use the terms bank defaults, bank failures and 'received assistance' interchangeably.

within two years of the first subsidiary default. We used the date of the first subsidiary default as the date of default for the holding company.[7]

2.2 Data Exclusions

Excluded Institutions

The goal of the RiskCalc model is to provide an Expected Default Frequency™ (EDF) for private banks.[8] The institutions covered in the model must have similar default characteristics. To create the most powerful model for this particular market, institutions that did not reflect the typical one in the market were eliminated. The following types of institutions have not been captured in the default data:

- **Credit unions** -- member-owned, nonprofit, financial institutions that serve circumscribed fields of membership and are exempt from all taxes imposed by the United States or by any state, territorial, or local taxing authority except for local real or personal property tax.
- **Noninsured institutions** -- including non-insured state-chartered and private banks, building and loan associations, personal loan companies, industrial banks, loan and investment companies, and similar institutions.
- **Non-deposit companies** -- including non-deposit trust companies and institutions chartered under banking or trust company laws, but operating as investment or title insurance companies and not engaged in deposit taking.
- **Foreign bank's branches** -- FDIC-insured domestic branches of foreign banks (IBA offices).
- **Central banks and similar institutions** -- Federal Reserve banks and other banks, such as the Federal Home Loan banks, which operate as rediscount banks and do not accept deposits except from financial institutions.
- **Publicly traded financial institutions** – For these institutions, we recommend the Moody's KMV Financial Firm Model for a more timely and accurate assessment of credit risk. Moody's KMV Financial Model takes advantage of the equity price information available for such institutions. The U.S. Banks model provides a "pass through" to this model for public firms.

2.3 Descriptive Statistics of the Data

Overview of the Data

The data on banks has increased substantially since the U.S. Banks v1.0 model was developed, in particular on the number of annual regulatory filings. Figure 1 presents the distribution of financial statements and defaults by year in our bank database. Table 1 summarizes the data used in the development, validation, and calibration of the U.S. Banks v3.1 model.

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[7] We experimented with weighting by the assets of the assisted subsidiary banks but found that it had little impact on default flags.

[8] To apply the model to private bank holding companies, it is required that consolidated financial statement information of parent holding companies be available, i.e. Y-9C report.

FIGURE 1 Date Distribution of Bank & Thrift Financial Statements and Default Data

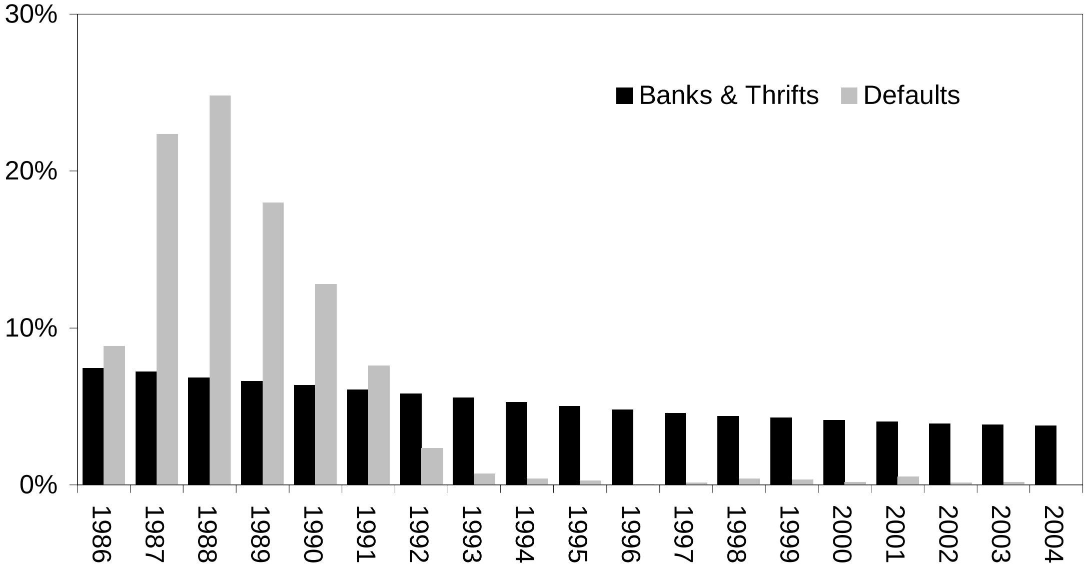

TABLE 1 Information on Bank Sample Data

FDIC-Insured Institutions	U.S. Banks v1.0 Model	U.S. Banks v3.1 Model	Bank Research Database Growth
Financial statements	161,034	242,003	↑50%
Unique number of Institutions	17,600+	21,200+	↑20%
Defaults	2,198	2,219	↑1%
Time period	1986-1999	1986-2004	↑ 5 additional years

TABLE 2 Information on the U.S. Banks v3.1 Model Sample Data[9]

	Banks	Thrifts	Bank Holding Companies with Consolidated Reports[10]
Financial statements	208,663	33,340	30,707
Unique number of Institutions	18,000+	3,000+	4,259
Defaults	1,187	1,032	115
Time period	1986-2004	1986-2004	1986-2004

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[9] The development sample includes annual observations on commercial banks and thrifts.

[10] Bank holding companies with consolidated assets greater than $150 million are required by the Federal Reserve Bank to submit consolidated report, i.e. Y-9C, on quarterly basis. The data count for the whole banking holding companies sample is 112,458 statement-year observations and 11,326 unique firms.

Robustness of the Data

In building a model, potential database weaknesses need to be examined. Not only does the database need to cover a large number of banks and defaults, but the defaults also need to be distributed among bank types covered. For example, if the database has significant numbers of small institutions or state-chartered institutions and there are not sufficient defaults in those areas, the model may not be a good default predictor. The sample used in developing the RiskCalc models has addressed both of these issues.

Figure 2 presents the distributions of defaults and banks by charter category. Figure 4 presents the same distributions by the size of banks as measured by total assets. These figures demonstrate that for commercial banks, the proportion of defaults in any one size group or charter group is roughly comparable to the proportion of institutions in these groupings. Thrifts tend to be larger than banks on average. In addition, for thrifts, there are proportionately more assisted and failed institutions at the larger end of the size distribution.

FIGURE 2 Distribution of Defaults for Banks by Chart Class

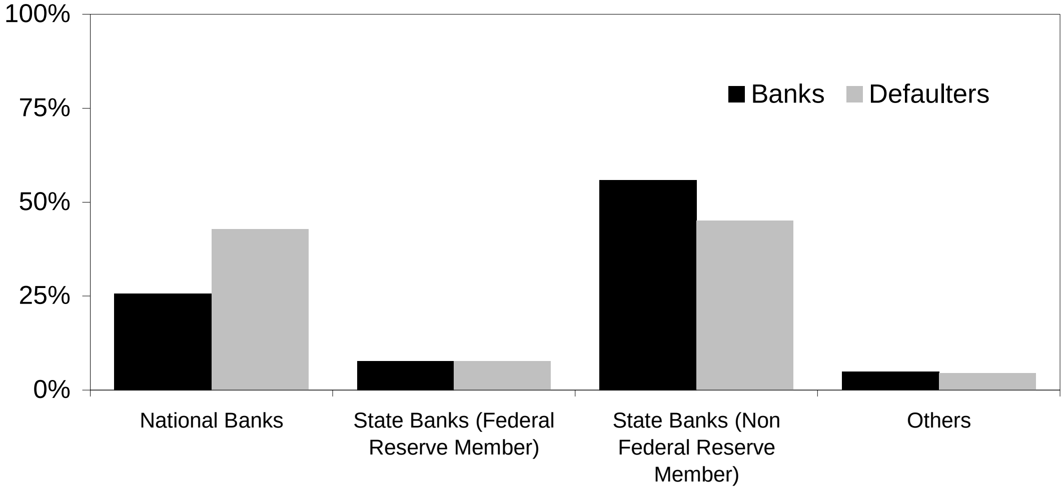


FIGURE 3 Distribution of Defaults for Thrifts by Chart Class

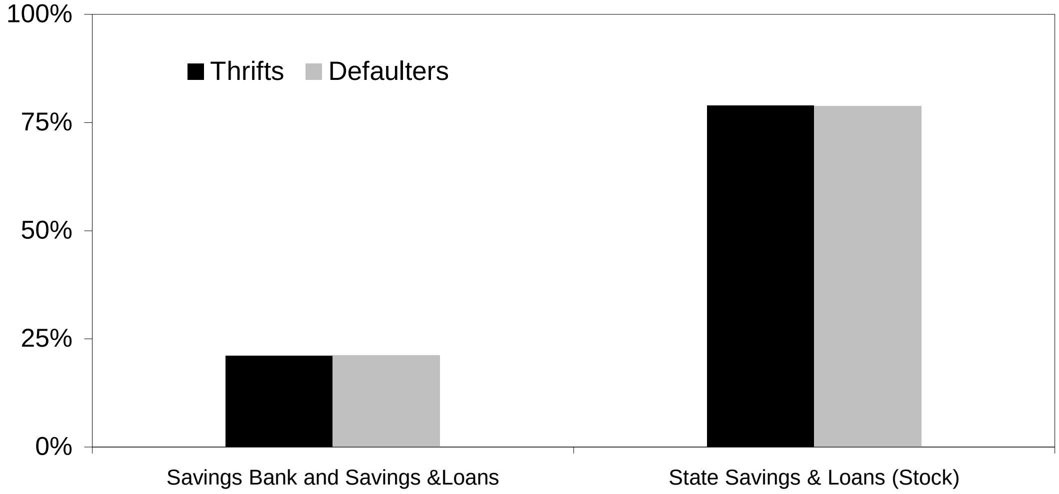


FIGURE 4 Size (as Total Assets) Distribution of Defaults for Banks

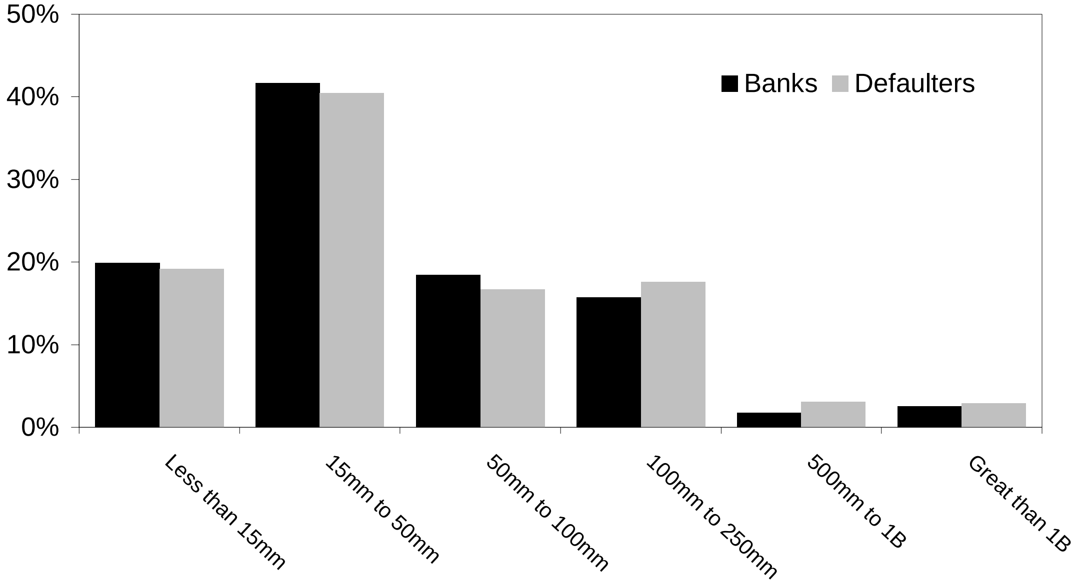
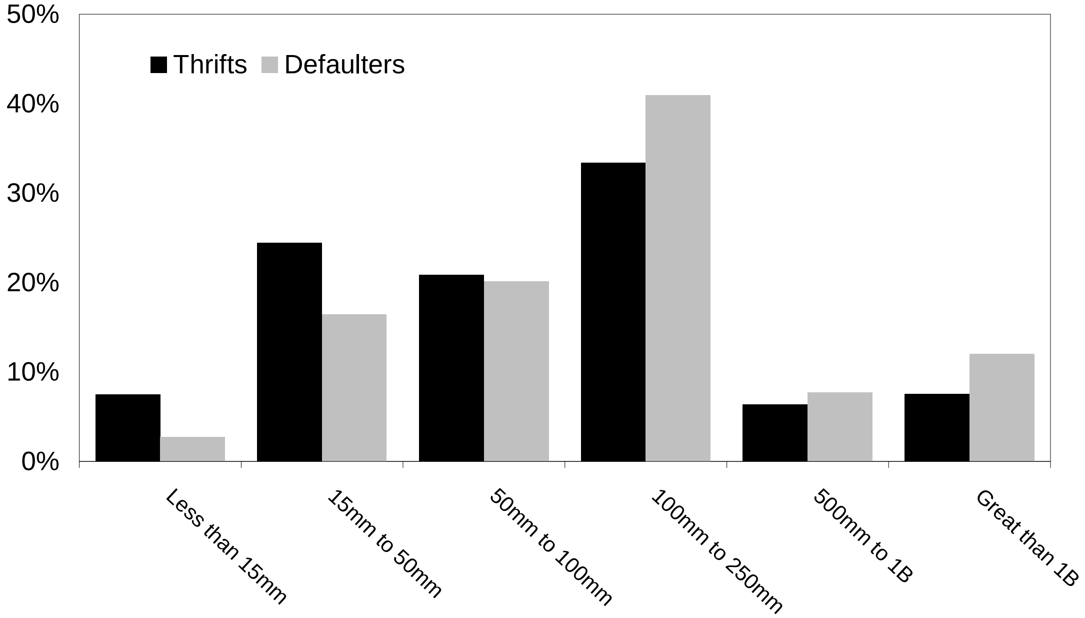


FIGURE 5 Size (as Total Assets) Distribution of Defaults for Thrifts



2.4 Cleaning the Data

In the development of a RiskCalc model, the first step is the collection of a large and appropriate database. In addition, data needs to be "cleaned" so that it is representative of the actual risk of the banks covered. Moody's KMV has developed techniques for cleaning the database to improve the model results.

2.5 Central Default Tendency

The estimate of long-run aggregate probabilities of default (or central default tendency) is important as an anchor for a model. The best estimation of default probability is a ratio that would reflect the number of obligors that defaulted in one year compared with the total obligors at the beginning of that year.

Since most banks do not default, bank defaults are relatively rare and thus more valuable in building a default prediction model. For U.S. private banks, the collection on bank failure & assistance data through FDIC is particularly systematic and consistent.[11] Nevertheless, the vast majority of the default events in the sample are associated with one specific banking crisis (see Figure 1). As a result, the sample default rate, a critical source to the estimate of central default tendency, is heavily influenced by the sample period chosen for the analysis (see Table 3).[12]

The estimate of the central default tendency for U.S. private banks is based on several different sample periods in the development data, which lead us to an estimate of 0.6% for the 1-year model for banks with assets greater than 100 million.[13] This estimate is consistent with the average probability of default from the U.S. Banks v1.0 model on the same universe. With this estimate, the average EDF credit measures of our sample are approximately 26% greater than the average of annual failure/assistance (F&A) rate for large banks and thrifts in 1986-2004 (see Table 3).[14] This upward adjustment incorporates a margin of conservatism into the model for the overall sample with the level of conservatism

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[11] The data not only provides complete historic records of FDIC resolution, but also tracks bank active/closing status throughout the time.

[12] Credit Suisse First Boston (2005) referred to the time period of 1992-2004 as the 'modern banking era'.

[13] The same universe was adopted by the U.S. Banks v1.0 model.

[14] The Rate is computed on a quarterly basis and is equal to the number of obligors that defaulted within one year divided by the total number of obligors at the beginning of the year.

much higher in the post banking crisis period. For thrifts, during the 1986-1991 period when the savings and loan crisis caused heightened default activity, the model tended to under-predict the default rate.[15] For banks, in the 13-year period of low default activity following the banking crises, the model over-predicts defaults by more than 100%. To summarize, the model is conservative across subsets of the data with the exception of thrifts during 1986-1991 where the model understates default risk by 22%.[16]

TABLE 3 Comparing Average EDF Credit Measures with Actual Default Rates

FDIC-Insured Entities (Assets > 100 Million)		**1986-1991**	**1992-2004**	**1986-2004**
Banks	Average Default Rate	1.26%	0.11%	0.45%
	Average EDFs	1.31%	0.27%	0.57%
	Relative Difference	↑4%	↑143%	↑26%
Thrifts	Average Default Rate	8.67%	0.41%	2.82%
	Average EDFs	6.80%	0.65%	3.59%
	Relative Difference	↓22%	↑59%	↑27%

Calculating a 5-year Central Default Tendency

There is a lack of publicly available data for direct calculation of the central default tendency rate of a cumulative 5-year default probability. Based on extensive Moody's KMV research, a 5-year cumulative default tendency is derived from the 1-year estimate. This research, combined with the information provided by the CRD, shows that the 5-year cumulative default rate is, on average, 4 times the level of the 1-year default rate. Therefore, 2.4% is used as the central default tendency for the 5-year model.

Central Default Tendency in FSO and CCA Modes

In the FSO mode, the central default tendency remains constant over time. In CCA mode, the central default tendency is equal to the central default tendency of the FSO mode when the effects of the credit cycle are neutral. When the forward-looking prediction of the credit cycle indicates increasing default risk, the central default tendency of the CCA mode will be larger, and when the effects of the credit cycle indicate reducing default risk, the central default tendency will be smaller.

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[15] According to the FDIC database, at the end of 1986 there were 3,589 active thrifts. At the end of 1990, this number dropped to 2,596. Most of the closings were triggered by failure.

[16] A properly calibrated model should over predict defaults most of the time due to the skewness of the distribution of defaults (cf., Kurbat and Korablev, 2002 and Dwyer, 2006). Nevertheless a large systematic shock (such as a banking crisis) can lead to default rates that exceed the predicted values during these years.

3 MODEL COMPONENTS

The U.S. Banks v3.1 model incorporates various components to determine the EDF credit measure. The inputs to the model include selection of the financial ratios and transforms of those ratios, and the credit cycle adjustment.

The development of a RiskCalc model involves the following steps:

1. Choosing a limited number of financial statement variables for the model from a list of possible variables.[17]
2. Transforming the variables into interim probabilities of default using non-parametric techniques.
3. Estimating the weightings of the financial statement variables, using a probit model combined with industry variables.
4. Creating a (non-parametric) final transform that converts the probit model score into an actual EDF credit measure.

In FSO mode, the models are based on the following functional form:

$$FSO\ EDF = F\left(\Phi\left(\sum_{i=1}^{N}\beta_i T_i(x_i) + \sum_{j=1}^{K}\gamma_j I_j\right)\right)$$

where $x_1,\ldots,x_N$ are the input ratios; $I_1,\ldots,I_K$ are indicator variables for each of the industry classifications; β and γ are estimated coefficients; Φ is the cumulative normal distribution; F and $T_1,\ldots,T_N$ are non-parametric transforms; and FSO EDF is the financial-statement-only EDF credit measure.[18] The Ts are the transforms of each financial statement variable, which capture the non-linear impacts of financial ratios on the default likelihood. (This is shown in Figure 6 on page 16 and discussed in detail later in the document.) F is the final transform (i.e., the final mapping). The final transform captures the empirical relationship between the probit model score and actual default probabilities. We describe the final transform as calibrating the model score to an actual EDF credit measure. The difference between the FSO EDF credit measure and the credit cycle adjusted EDF credit measure is that in CCA mode the final transform is adjusted to reflect our assessment of the current stage of the credit cycle, while in FSO mode it remains constant.

3.1 Financial Statement Variables

Selecting the Variables

Our variable selection process starts with a long list of possible financial statement variables. The working list of ratios is divided into groups that represent different underlying concepts regarding a bank's financial status (see Table 4). A model is then built with at least one variable per group. When it is possible to increase model performance while maintaining model robustness, several variables from each group will be used in the model. We ask the following questions when deciding which variables to include in the final model:

- Is the variable readily available?
- Are the definitions of the inputs to the variable unambiguous?
- Is the meaning of the variable intuitive?
- Does the variable predict default activity?
- Is the variable generally uncorrelated with other variables in the model?

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[17] These variables are often ratios, but not always. For example, one measure of profitability is Net Income to Total Assets, which is a ratio, and one measure of size is total assets, which is not a ratio.

[18] By non-parametric, we mean that the $T(x_i)$ is a continuous function of x not requiring a specification of a specific closed (or parametric) functional form. We estimate these transforms using a variety of local regression and density estimation techniques.

TABLE 4 Groupings of Potential Risk Drivers for Banks

Examples of ratios in the **capital structure** group include liabilities or equity capital to assets. → *High leverage increases the probability of default.*

Examples of ratios in the **profitability** group include: net income and net interest income in the numerator; and total assets, average earning assets in the denominator. → *High profitability reduces the probability of default.*

Asset concentration ratios include commercial & industrial loans to assets, loans secured by commercial real estate to assets, etc. These ratios measure the extent to which a bank has a substantial portion of loan assets in accounts that may be exposed to business segments with high volatility. → *High concentration on volatile assets increases the probability of default.*

Liquidity variables include variants of liquid securities to assets, e.g., treasury securities to assets, treasury securities plus municipal securities to assets. These variables measure the extent to which the bank has liquid assets relative to the size of its total assets. → *High liquidity reduces the probability of default.*

Asset quality ratios include commercial loan charge-offs to assets, consumer loan charge-offs to assets and other real estate owned to assets.[19] These variables measure the extent of losses incurred by the banks, which reflects the quality of loan assets held by banks. → *Low asset quality increases the probability of default.*

TABLE 5 Financial Statement Variables used in the U.S. Banks v3.1 Model

Category	Variable
Capital Structure	**Total Equity Capital to Assets:** Equity Capital to Total Assets
Profitability	**Net Income to Assets:** Net Income to Total Assets
	Net Interest Margin: Net Interest Margin
Asset Concentration	**Concentration Risk:**, Commercial Real Estate Loans Construction Loans and Commercial & Industrial Loans to Total Assets
Liquidity	**Government Securities, Muni and MBS to Assets:** Treasury Securities, Securities Sponsored by Government & Government Agency, Municipal Securities and MBS to Total Assets
Asset Quality	**Commercial Loan Charge-Offs to Assets:** Commercial Loan Charge-Offs to Total Assets
	Consumer Loan Charge-Offs to Assets: Consumer Loan Charge-Offs to Total Assets
	Other Real Estate Owned to Assets: Other Real Estate Owned to Total Assets

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[19] Other Real Estate Owned is a line item on a bank's balance sheet. This position excludes real estate actually owned by banks for conducting normal business. It mainly includes (1) real estate acquired in any manner for debts previously contracted, e.g. real estate acquired through foreclosure; or (2) real estate collateral underlying a loan when the bank has obtained possession of the collateral.

Variable Transforms

Once the variables are selected, they are transformed into a preliminary EDF credit measure. Figure 6 presents the transformations used in the model. The horizontal axis gives the percentile score of the ratio and the vertical axis gives the default probability of that ratio in isolation (univariate). The percentile score represents the percent of the database that had a ratio below that of the company (e.g., if liquidity is in the 90th percentile that means that 90% of the sample had lower liquidity than that bank).

The shape of the transformation indicates how significantly a change in level impacts the EDF credit measure. If the slope of the transform is steep, a small change will have a larger impact on risk than if the slope were flat.

For the **Capital Structure** group, the transform for Equity Capital to Assets is downward-sloping. Substantial equity corresponds to low leverage and low default risk. When leverage is high, the slope of the transform is much steeper than when leverage is at medium and low levels. This indicates that level of equity ratio is not a strong signal of bank distress until it reaches the low percentile region (Figure 6).

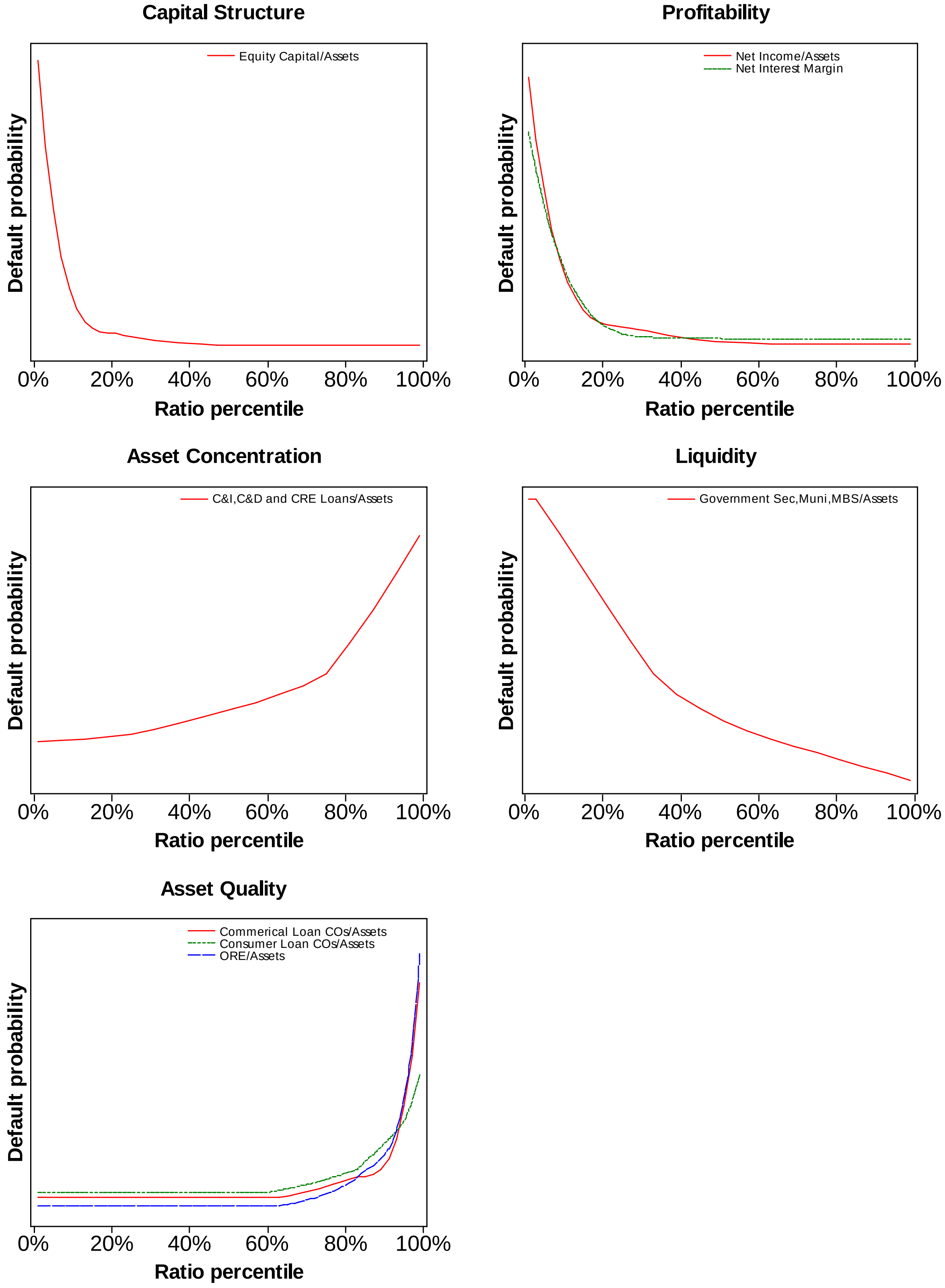
For the **Profitability** group, the transforms for Net Income to Assets and Net Interest Margin are both downward-sloping. Similar to the capital structure ratio, profitability ratios have much flatter slope as they grow large (Figure 6), which suggests that low profitability makes default more likely, but it becomes much less indicative when the bank's earning capacity surpasses the 20th percentile.

The **Asset Concentration** variable is Commercial & Industrial Loans, Commercial Real Estate Loans and Construction Loans to Asset. High levels of this ratio correspond to a high concentration of loans that are amongst the most volatile in the portfolio. The slope of the transform increases as the ratio increases, indicating that sensitivity to default risk increases as the ratio increases (Figure 6).

For the **Liquidity** group, the transform for Government Securities to Assets is downward-sloping, which means the more liquidity a bank has, the lower default risk (Figure 6). The slope of the transform decreases slightly as the ratio decreases.

The **Asset Quality** variables are [Commercial Loan Charge-Offs to Assets], [Consumer Loan Charge-Offs to Assets] and [Other Real Estate Owned to Assets]. These variables are all upward-sloping. Large values of asset quality variables correspond to high recognized or potential losses to the loan assets. Therefore, default risk rises as these three variables increase. The slopes of all three transforms are very flat until the median level and become very steep after the 80th percentile (Figure 6).

FIGURE 6 Transformations of Financial Statement Variables Used in the Model



3.2 Model Weights

Importance

The relative value of each variable used in calculating an EDF credit measure is important in understanding a company's risk. The non-linear nature of the model makes the weight of the variables more difficult to determine since the actual impact on the risk depends on the coefficient, the transformation shape, and the percentile ranking of the company. The model weights, therefore, are calculated based on the average EDF value for the transformation and its standard deviation. A variable with a flat transformation could have a low weight, even if the coefficient is large (Figure 6).

Calculation of Weights

To calculate the weighting of a variable, the EDF credit measure for a theoretical bank with all its variables at the average transformation values is computed. The variables are then increased one at a time by one standard deviation. The EDF change for each variable (in absolute value) is computed and added together. The relative weight of each variable is then calculated as the EDF level changes for that variable as a percent of the total change in EDF level. This gives the variable with the biggest impact on the EDF level the biggest weight, and the variable that has the smallest impact on the EDF level the smallest weight. Since the weights are a percentage of the total EDF value, they sum to 100%. The weight of each category is the sum of the weights of each variable in that category. Table 6 presents the weights in the U.S. Banks v3.1 model. The two most important categories are capital structure and profitability.

TABLE 6 Risk Drivers in the U.S. Banks v3.1 Model

Category	Weights
Capital Structure Total Equity Capital to Assets	28%
Profitability Net Income to Assets Net Interest Margin	24%
Asset Concentration Concentration Risk	23%
Asset Quality Commercial Loan Charge-Offs to Assets Consumer Loan Charge-Offs to Assets Other Real Estate Owned to Assets	19%
Liquidity Government Securities, Muni and MBS to Assets	7%

3.3 Credit Cycle Adjustment

EDF credit measures are impacted not only by the financials of a company, but also by the general credit cycle in the economy. To capture this effect, the U.S. Banks v3.1 model includes a credit cycle adjustment factor. The credit cycle adjustment is designed to incorporate the current credit cycle into the estimate of private bank default risk.

Selecting an Adjustment Factor

The U.S. Banks v3.1 model uses the distance-to-default calculation from the Moody's KMV Financial Firm Model. This measure is specifically designed to be a forward-looking indicator of default risk. It extracts signals of default risk from the stock market performance of individual firms (cf. Bohn and Crosbie, 2003 and Arora and Sellers, 2005).

If the distance-to-default for public firms in the banking industry indicates a level of risk above the historical average, then the private banks' EDF credit measures are adjusted upward. Conversely, if the level of risk is below the historical average, then the private banks' EDF credit measures are adjusted downward. When the credit cycle adjustment factor is neutral, the CCA EDF credit measure coincides with the FSO EDF credit measure.

Adjustment Factor used in the Model

For the U.S. Bank model, the distance-to-default factor is an index based on the individual public banking firms' distance-to-default series. When selecting the impact of the distance-to-default factor on EDFs in the model, we carefully considered changes in the regulatory environment and advances in risk management tools over time. There were two recessionary periods in the sample that had very different impacts on the private banking industry. Figure 5 shows the relationship between private bank defaults and the corporate speculative grade bond default rate. Throughout history, increases in corporate defaults have not necessarily coincided with increases in bank failures.

In the late 1980s there was a banking crisis during the recessionary period. This was primarily driven by a combination of factors. Banks carried high leverage and low profitability into the recession that coincided with a deterioration of the real-estate markets in the U.S.. Figures 7 and 8 show the behavior of leverage and profitability over time and highlight negative profits and low leverage during the banking crisis. Figure 9 shows the variation of foreclosed real estate properties over time which peaked during the banking crisis.

The recessionary period in the early 2000s was accompanied by an increase in the public bank default risk as measured by the distance-to-default factor (Figure 6) along with the speculative grade default rate (Figure 5). However, leverage levels and profitability were much stronger in this period for private banks leading to lower rates of bank failures. These facts, in combination with a robust real estate market throughout the recession (Figure 11), lead to much lower levels of EDF for this period, even though the credit cycle adjustment was increasing risk. Therefore, the models average EDF for banks tracks the level of private bank defaults well.

Schuermann (2004) discusses differences in the banking environment in the two most recent recessionary periods. Given changes in regulation, banks were better able to diversify risk geographically and across business lines. In addition, banks are able to get a higher return for bearing risk by charging higher spreads for riskier loans. This combination of factors allowed bank profitability to remain high when risk levels increased in the most recent recession.

FIGURE 7 U.S. Bank Failures and Speculative Grade Default Rate: 1934-2004

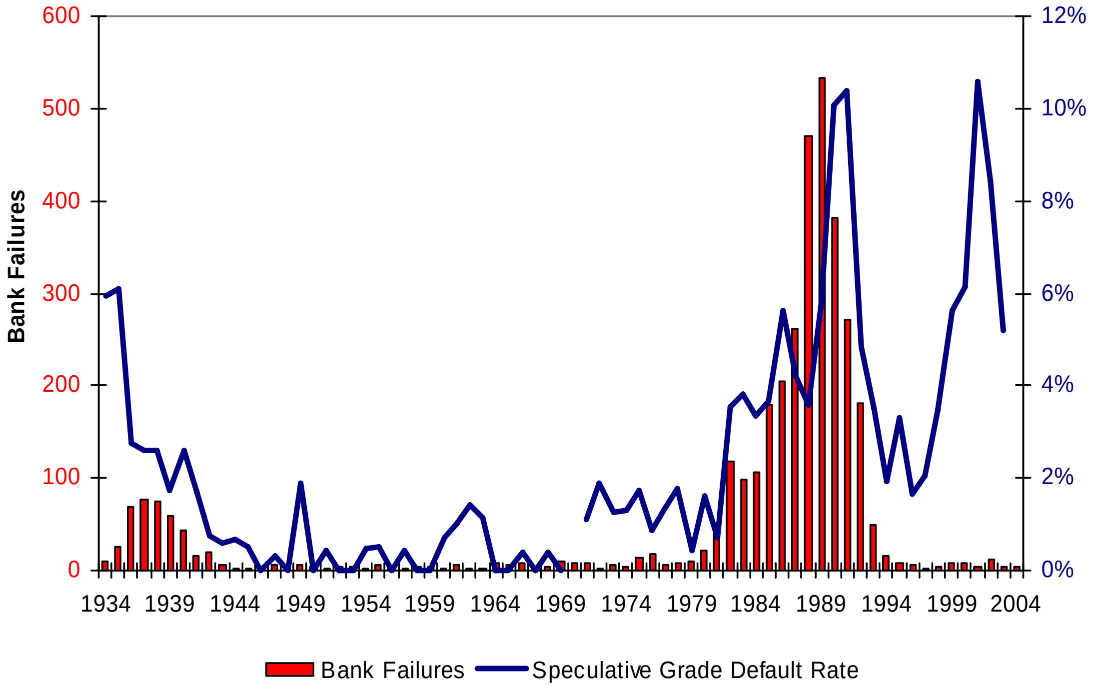
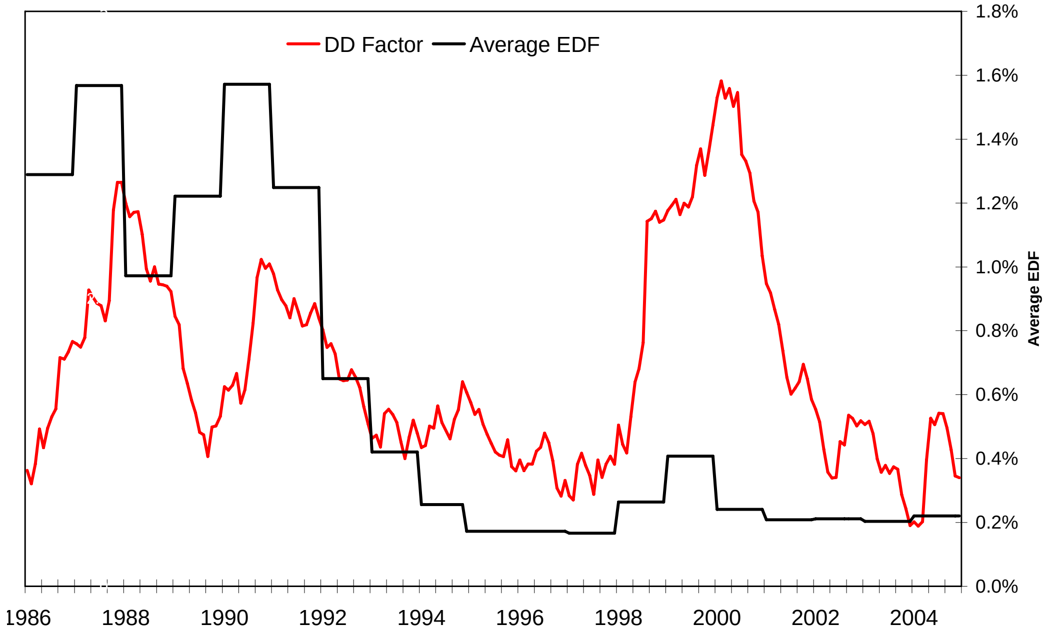


FIGURE 8 Distance to Default Factor and Average EDFs: 1986-2004*



*DDfactor has been inverted so that high levels indicate elevated risk.

FIGURE 9 Equity to Assets of Banks 1986-2005

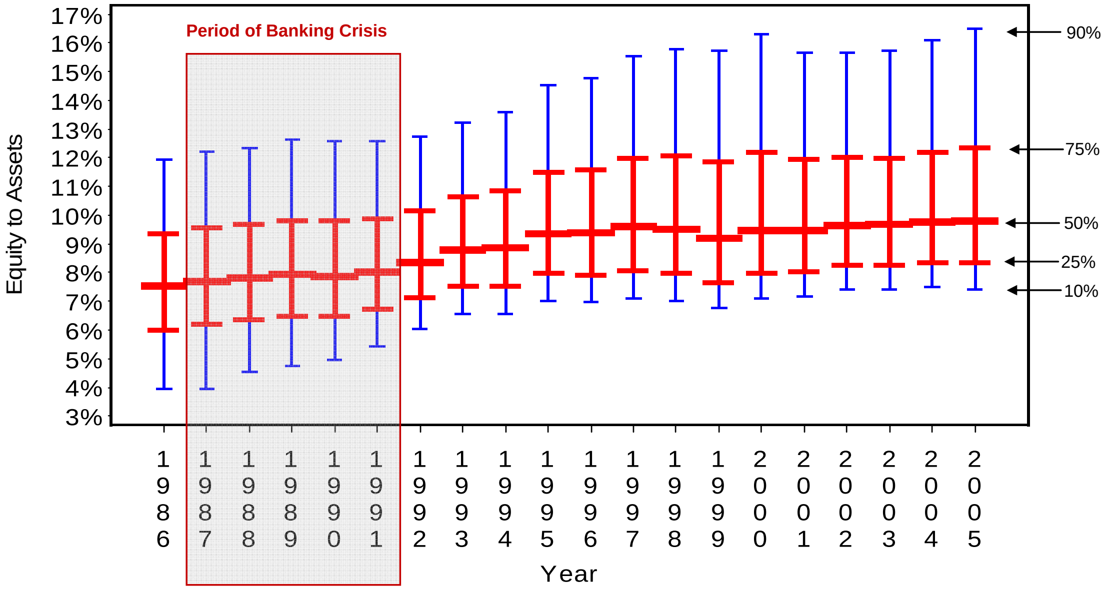


FIGURE 10 Net Income to Assets of Banks 1986-2005

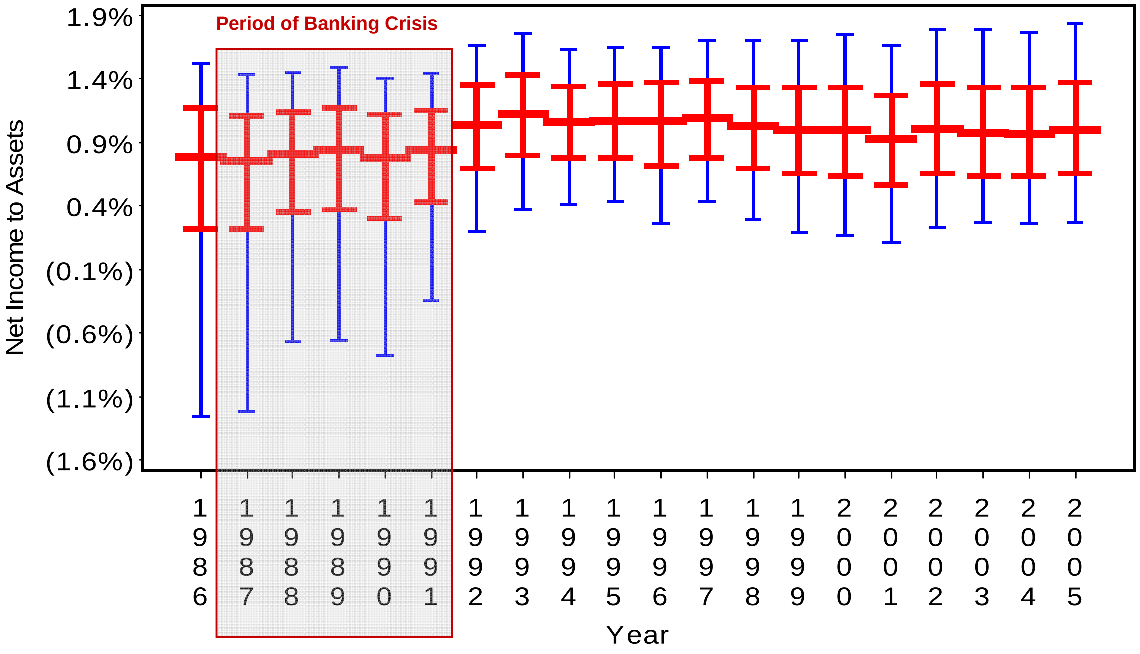


FIGURE 11 Other Real Estate Owned to Assets of Banks 1986-2005

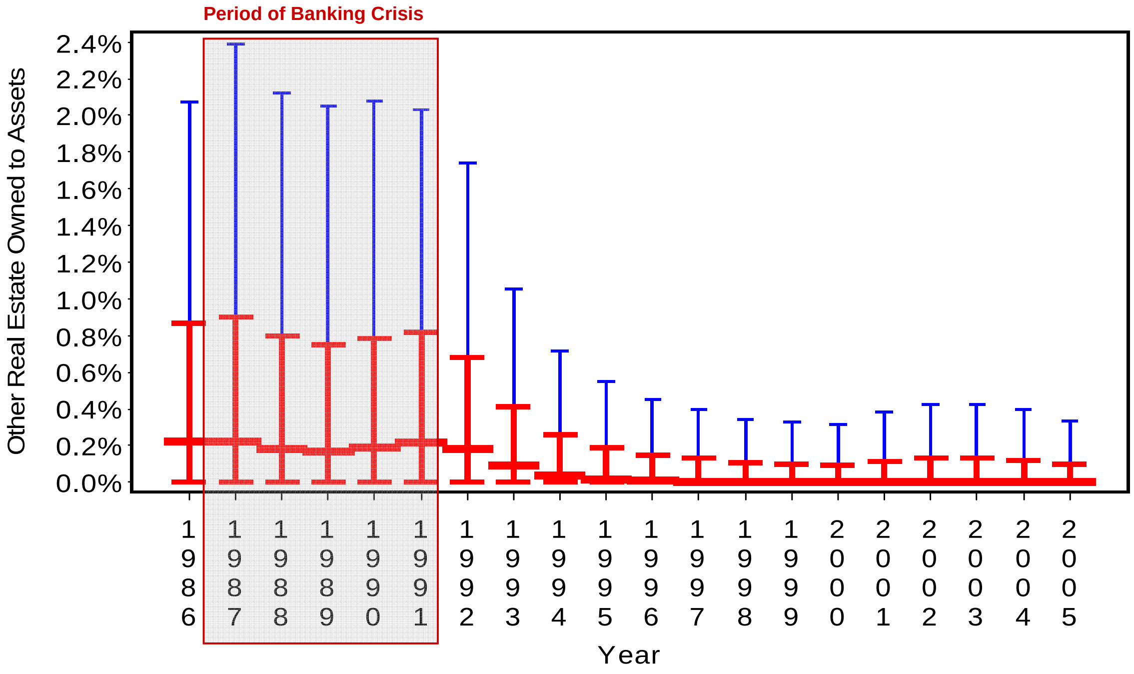
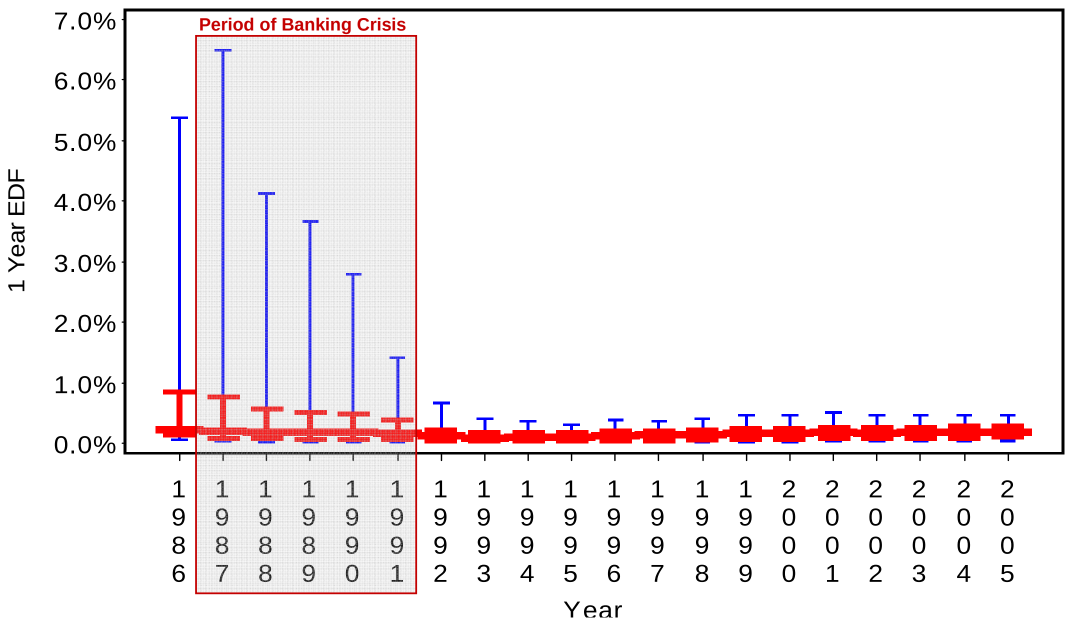


FIGURE 12 1 Year EDFs of Banks 1986-2005



4 VALIDATION RESULTS

Once a model is developed, it must be shown to be effective in predicting defaults. In this section, we present testing results on the model's ranking power (the model's ability to sort credits from worst to best) and the accuracy of its predicted EDF credit measure (the model's ability to estimate correctly the level of EDF).

The tests need to check not only the model's effectiveness, but also its robustness and how well it works on data outside the sample. To do out-of-sample testing, we performed walk-forward and *k*-fold analyses. The results of the testing show that the model is uniformly more powerful than other models across different time periods, sectors, and size classifications.

4.1 Increase in Overall Model Power and Accuracy

Table 7 presents the in-sample overall measures of power and likelihood for the U.S. Banks v3.1 model versus the U.S. Banks v1.0 model.[20] The U.S. Banks v3.1 model outperformed the U.S. Banks v1.0 model by 1.3 points and 2 points at the 1-year and 5-year horizons respectively.[21]

The reader may notice that the accuracy ratios are very high for both U.S. Banks models relative to other credit risk models—especially at the 1-year horizon. The high levels of accuracy are a consequence of the transparency of a banks financial statements. For example, as bank equity falls below a certain point, failure becomes almost inevitable. A corporate entity, in contrast, can have negative equity and not be in danger of defaulting.

TABLE 7 Power Enhancements of the U.S. Banks v3.1 Model

	One-year Model	**Five-year Model**
	Accuracy Ratio	**Accuracy Ratio**
U.S. Banks v3.1 Model	95.3%	85.0%
U.S. Banks v1.0	94.0%	83.0%
P-Value[22]	<0.001	<0.001

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[20] The sample includes banks and thrifts but not bank holding companies.

[21] The corresponding accuracy ratios for FSO are 95.1% for the one-year horizon and 84.3% for the five-year horizon.

[22] Indicates a rejection of the hypothesis is that difference of the v3.1 AR from the v1.0 AR is zero with over 99.9% confidence.

FIGURE 13 Power of Alternative Models (1-year and 5-year) — U.S. Banks

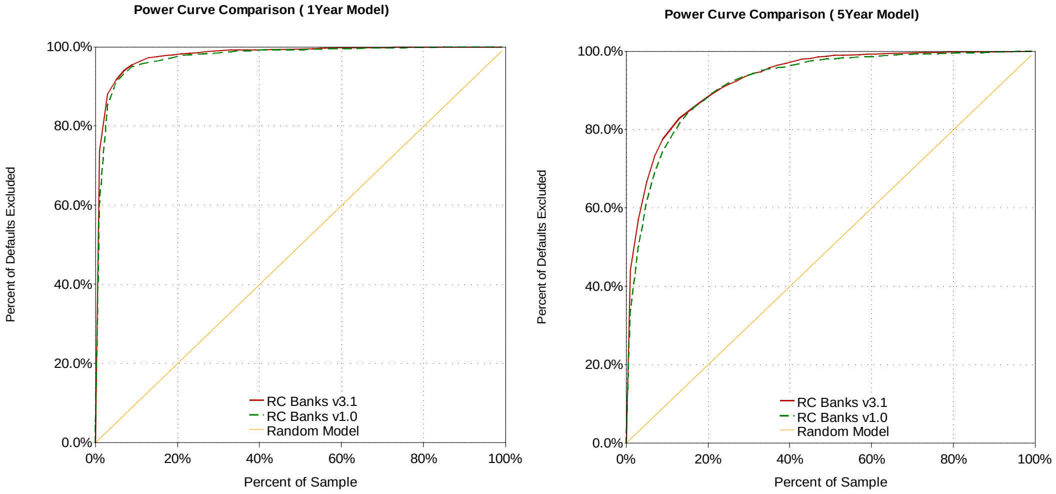

Figure 13 presents the cumulative accuracy profiles for the 1- and 5-year models corresponding to Table 7.

4.2 Correlations and Variance Inflation Factors

To ensure model robustness, the model must be tested for excessive multicollinearity, which occurs if a number of the variables used in the model are highly correlated. Excessive multicollinearity can cause instability in parameter estimates. To check for this issue, the correlation coefficients (Table 8) for the financial statement ratios in the model and the variance inflation factors (Table 9) are computed on the transformed variables (see Figure 6).

Model Results

The highest correlation coefficient is between [Equity Capital/Total Assets] and [Net Income/Total Assets] (46%). The next highest coefficient is between [Equity Capital/Total Assets] and [Net Interest Margin] and [Equity Capital/Total Assets] and [Other Real Estate Owned/Total Assets] (45%). Such coefficients are below what we would typically consider indications of multicollinearity. This finding is also verified by the VIF analysis.

TABLE 8 Correlations Among the Transformed Input Factors (Spearman Rank)

	Net Income to Assets	Total Equity Capital to Assets	Net Interest Margin	Commercial Loan Charge-Offs to Assets	Other Real Estate Owned to Assets	Concentration Risk	Government Securities, Muni and MBS to Assets	Consumer Loan Charge-Offs to Assets
Net Income to Assets	1.00							
Total Equity Capital to Assets	0.46	1.00						
Net Interest Margin	0.36	0.45	1.00					
Commercial Loan Charge-Offs to Assets	0.41	0.23	-0.04	1.00				
Other Real Estate Owned to Assets	0.41	0.45	0.19	0.31	1.00			
Concentration Risk	0.10	0.02	-0.15	0.08	0.09	1.00		
Government Securities, Muni and MBS to Assets	0.20	0.15	0.18	0.03	0.12	0.26	1.00	
Consumer Loan Charge-Offs to Assets	0.16	0.11	-0.09	0.23	0.11	-0.04	0.07	1.00

The Variance Inflation Factors (Table 9) for the financial statement variables represent how much of the variation in one independent variable can be explained by all the other independent variables in the model. The correlation coefficient, however, measures only the relationships between two variables. As shown in Table 9, the estimated VIF values in the Bank Model are notably below the threshold levels of 4 to 10 that are commonly used in VIF analysis when testing for presence of multicollinearity.[23] These findings indicate that the model variables do not present any substantial multicollinearity.

TABLE 9 Variance Inflation Factors

Variable	VIF
Net Income to Assets	1.67
Total Equity Capital to Assets	1.58
Net Interest Margin	1.50
Commercial Loan Charge-Offs to Assets	1.36
Other Real Estate Owned to Assets	1.35
Concentration Risk	1.16
Government Securities, Muni and MBS to Assets	1.15
Consumer Loan Charge-Offs to Assets	1.10

4.3 Model Power by Bank Characteristics and Size Groups

It is important to test the power of a model not only overall, but also among different institution type group and bank sizes. Table 10, Table 11, and Table 12 present the power comparisons by institution type group for both the 1-year and 5-year models.

The U.S. Banks v3.1 model outperforms the U.S. Banks v1.0 model in all three groups. Between bank and thrift groups, the one-year model has higher power in the bank group while the five-year model tends to do better in the thrift group.

TABLE 10 Model Power by Characteristics Group: 1-year Model

FDIC-Insured Entities	Percentage of Defaults	AR* RiskCalc v3.1	AR RiskCalc v1.0
Banks	53.7%	94.7%	93.7%
Thrifts	46.3%	92.1%	89.9%

*AR = accuracy ratio

TABLE 11 Model Power by Characteristics Group: 5-year Model

FDIC-Insured Entities	Percentage of Defaults	AR RiskCalc v3.1	AR RiskCalc v1.0
Banks	53.3%	80.9%	80.3%
Thrifts	46.7%	83.9%	79.4%

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[23] As Woolridge (2000) shows, VIF is inversely related to the tolerance value ($1-R^2$), such that a VIF of 10 corresponds to a tolerance value of 0.10. Clearly, any threshold is somewhat arbitrary and depends on the sample size. Nevertheless, if any of the R^2 values are greater than 0.75 (so that VIF is greater than 4.0), we would typically suspect that multicollinearity could be a problem. If any of the R^2 values are greater than 0.90 (so that VIF is greater than 10) we then conclude that multicollinearity is likely to be a serious problem.

TABLE 12 Model Power on Bank Holding Companies Group

Bank Holding Companies (Post 1989 Sample)[24]	**Percentage of Defaults of Whole BHCs Sample**	**AR RiskCalc v3.1**	**AR RiskCalc v1.0**
1 Year Model	42.5%	95.1%	94.8%
5 Year Model	40.8%	92.1%	92.0%

Table 13 and Table 14 present the power comparisons by bank size for the 1-year and 5-year models, respectively. The U.S. Banks v3.1 model outperforms the U.S. Banks v1.0 model consistently in all size groups. The highest power in the 1-year model is found in the banks with total assets between 100 millions to 500 millions, and the lowest is in the smallest banks—less than $100 million in assets. The highest power in the 5-year model is also found in the banks with total assets between $100 million to $500 million, and the lowest is in the banks with assets between $500 million and $1 billion.

TABLE 13 Model Power by Size: 1-year Model

Bank and Thrifts	**Percentage of Defaults**	**AR RiskCalc v3.1**	**AR RiskCalc v1.0**
Less than 100mm	59.3%	94.8%	93.8%
100mm - 500mm	28.3%	96.7%	95.4%
500mm -1 billion	5.3%	95.1%	92.6%
Greater than 1 billion	7.1%	95.3%	92.0%

TABLE 14 Model Power by Size: 5-year Model

Bank and Thrifts	**Percentage of Defaults**	**AR RiskCalc v3.1**	**AR RiskCalc v1.0**
Less than 100mm	57.1%	84.5%	83.3%
100mm - 500mm	29.7%	87.0%	84.9%
500mm -1 billion	6.4%	83.0%	77.3%
Greater than 1 billion	6.8%	83.2%	71.8%

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[24] The bank holding companies data did not include commercial real estate loans, and construction and land development loans as sub-items of loans secured by real estate before the 1990s.

4.4 Power Performance Over Time

Since models are implemented at various points in a business cycle by design, power tests by year (Table 15 and Table 16) were conducted to examine whether the model performance is excessively time dependent.

Table 15 and Table 16 present the results from this analysis at the 1- and 5-year horizons, respectively. The accuracy ratio of the U.S. Banks v3.1 model is compared with the U.S. Banks v1.0 model for each year. As shown in these tables, The U.S. Banks v3.1 model consistently outperforms the U.S. Banks v1.0 model at the 1-year horizon and the margin improves in recent periods. At the 5-year horizon, the pattern persists except for 1992–1993 period.

TABLE 15 Model Power Over Time: 1-year Horizon

Bank and Thrifts	Percent of Defaults	AR* RiskCalc v3.1	AR RiskCalc v1.0
1986 – 1987	42.0%	89.5%	87.4%
1988 – 1989	35.4%	95.0%	92.4%
1990 – 1991	17.3%	94.6%	93.1%
1992 – 1993	2.7%	94.4%	93.5%
1994 – 1996	0.6%	98.0%	98.0%
1997 – 1999	0.9%	77.0%	70.9%
2000 – 2004	1.0%	77.5%	75.7%

*AR = accuracy ratio

TABLE 16 Model Power Over Time: 5-year Horizon

Bank and Thrifts	Percent of Defaults	AR RiskCalc v3.1	AR RiskCalc v1.0
1986 – 1987	52.7%	78.1%	75.3%
1988 – 1989	31.2%	85.3%	80.6%
1990 – 1991	11.6%	89.5%	87.5%
1992 – 1993	1.9%	85.8%	86.5%
1994 – 1996	0.9%	62.3%	55.8%
1997 – 1999	1.1%	42.6%	36.6%
2000 – 2002	0.6%	66.0%	58.0%

4.5 Out of Sample Testing: K-fold Tests

The model exhibits a high degree of power in distinguishing good credits from bad ones (in Table 7), but whether this power is attributable to the overall model effectiveness or the impact of a particular sub-sample also needs to be tested. A standard test for evaluating this is the "*k*-fold test," which divides the defaulting and non-defaulting companies into *k*-equally sized segments. This yields *k*-equally sized observation sub-samples that exhibit the identical overall default rate and are temporally and cross-sectionally independent. The model is then run on *k-1* sub-samples and these parameter estimates are used to score the *k-th* sub-sample. We repeat this procedure for all possible combinations, and put the *k*-scored "out-of-sample" sub-samples together to calculate an accuracy ratio on this combined data set.

Results

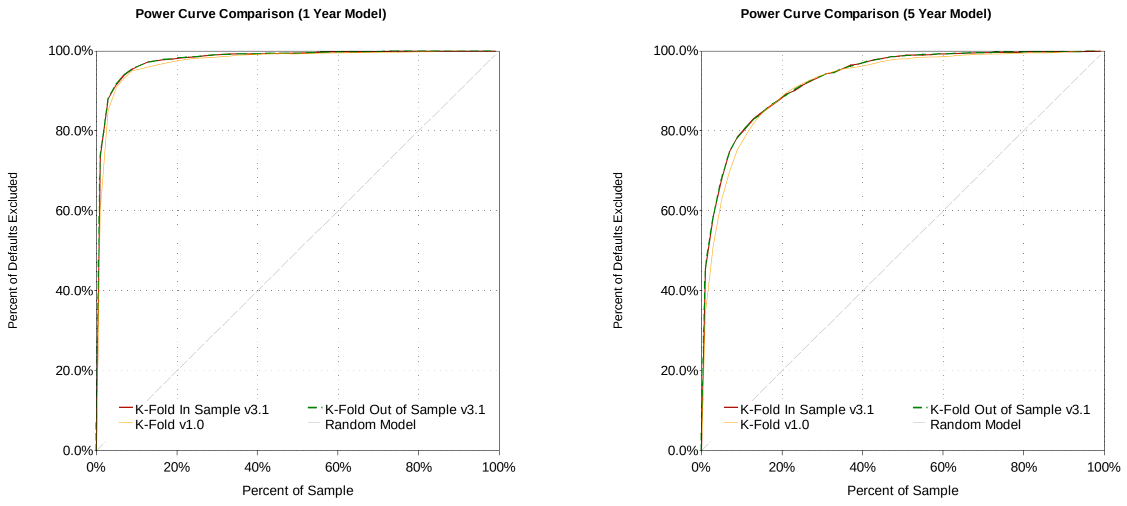
Table 17 summarizes the *k*-fold test results (with *k*=5). The reported figures are the accuracy ratios by the corresponding sample and time horizons. The out-of-sample model consistently out-performs the U.S. Banks v1.0 model. Figure 14 presents the cumulative accuracy profiles associated with the overall "out-of-sample" results against the in-sample results. The model performance is maintained both in- and out-of-sample in the *k*-fold analysis: the in- and out-of-sample power curves are indistinguishable.

TABLE 17 U.S. Banks v3.1 Model K-fold Test Results

	Out of Sample AR*		**RiskCalc v1.0**	
	1-year AR	**5-year AR**	**1-year AR**	**5-year AR**
Subsample 1	95.3%	88.2%	94.0%	85.6%
Subsample 2	94.8%	88.0%	94.4%	86.3%
Subsample 3	95.0%	87.8%	93.3%	85.4%
Subsample 4	95.1%	87.5%	93.4%	84.9%
Subsample 5	96.5%	89.3%	95.1%	86.5%
K-fold Overall	95.3%	85.0%	94.0%	83.0%
In-sample AR	95.3%	85.0%		

*AR = accuracy ratio

FIGURE 14 U.S. Banks v3.1 Model K-fold



The *k*-fold testing does not control for time dependence. Each of the *k* sub-samples contains data from all periods. As a result, if there were a particularly high period of default rates, this would be included in each of the *k* samples. Such testing does not give a true sense of the how the model would have performed during those volatile periods because the model is estimated with full information on those time periods.

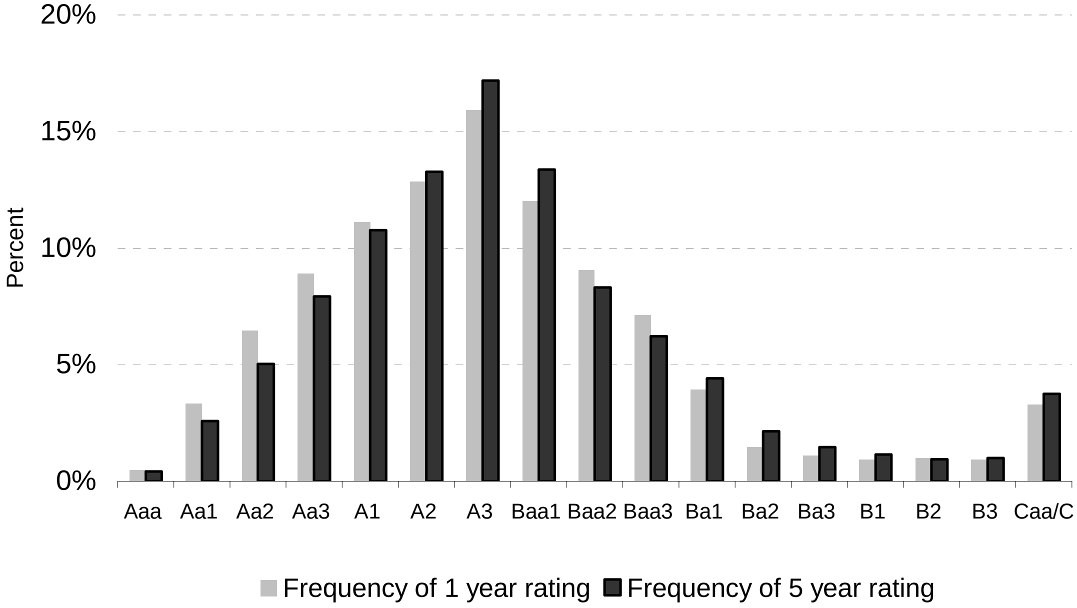
4.6 Model Calibration and Implied Ratings

To aid in the interpretation of an EDF credit measure, an EDF credit measure is mapped to a .edf rating (an EDF-implied rating). All RiskCalc v3.1 models to date have used the same mapping. This mapping is designed so that:

- There is a large range of .edf ratings (as required for economic and regulatory applications);
- No one rating contains too many credits (as required for economic and regulatory applications);
- The distribution of the 5-year ratings is approximately the same as the distribution of 1-year ratings (for consistency with rating-based analysis applications);
- The EDF credit measure associated with a .edf rating is approximately the same as the observed historical default rate associated with a Moody's bond rating (for consistency with rating-based analysis applications).

Figure 15 shows the distribution of bank data observations by rating category in the development sample (for the Financial Statement Only EDF credit measures over the full time period).[25] Note that 17 categories between Aaa and C/Caa are all utilized and that less than 20% of the observations are in each category. The 1-year and 5-year distributions peak at A3. While not reported here, other research has shown that the distribution of the CCA EDF implied ratings change over time with the credit cycle, while the distribution of the FSO EDF implied ratings remains relatively stable over time. We also carefully considered the differences in implied ratings between the v3.1 and v1.0 models. For private banks, a large percentage of v3.1 implied ratings fall within one to two notches of the v1.0 model implied rating.

FIGURE 15 EDF-implied Ratings for the 1- and 5-year Models in the Development Sample



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[25] For FDIC-insured entities with assets greater than 100 millions.

5 FURTHER MODEL IMPROVEMENTS

In this section, we briefly outline some other enhancements to the model. For a detailed discussion of these enhancements, refer to the Technical Document.

5.1 Continuous Term Structure

The previous version of the RiskCalc model provided the user with two discrete default probability estimates: a 1-year and a 5-year EDF. In this version, utilizing the two point estimates for 1- and 5-year estimates we fit a Weibull function, and thus achieve a continuous term structure of EDF credit measures for each credit. In other words, users of the U.S. Banks v3.1 model now can obtain EDF estimates for any point between 1 and 5 years[26]. In addition, RiskCalc v3.1 provides EDF estimates for alternative definitions, such as the Forward EDF and the Annualized EDF (Table 18):

- Cumulative EDF

  A cumulative EDF credit measure gives the probability of default over that time period. For example, a five-year cumulative EDF of 13.44% means that that company has a 13.44% chance of defaulting over that five-year period. The second column of Table 18 provides an example of the cumulative 1- to 5-year credit measures produced by the model.

- Forward EDF

  The forward EDF is the probability of default between t-1 and t conditional upon survival until t-1. In other words, the 4-year Forward EDF is the probability that a bank will default between years three and four assuming the bank survived to year 3.[27] The third column of Table 18 displays the forward 1- to 5-year EDF credit measures that are derived from the cumulative EDF credit measures.

- Annualized EDF

  The annualized EDF credit measure is the cumulative EDF credit measure for a given period, stated on a per year basis. For example, a company with a cumulative 5-year EDF of 13.44% would have a 5-year annualized EDF of 2.84%.[28] This means that the average default rate per year for a 13.44% cumulative default rate is 2.84%. The last column of Table 18 presents the annualized EDF credit measures for years 1 to 5 that are derived from the cumulative EDF credit measures.

TABLE 18 Term Structure of EDF Credit Measures: An Example

EDF	Cumulative	Forward	Annualized
Year 1	4.23	4.23	4.23
Year 2	7.00	2.90	3.57
Year 3	9.37	2.55	3.23
Year 4	11.49	2.34	3.01
Year 5	13.44	2.20	2.84

---

[26] We measured the relationship between forward EDF credit measures and the realized EDF from the next year using the sample data. The 1-year ahead forward EDF generally tracks the observed EDF well. During the banking crisis period we observed the realized EDF credit measures were slightly higher than the forward EDF credit measures. However, in the most periods the 1-year ahead forward EDF is a slightly conservative estimate of the realized EDF.

[27] Specifically, $FEDF_{t-1,t} = (CEDF_t - CEDF_{t-1})/(1-CEDF_{t-1})$, where $FEDF_{t-1,t}$ is the forward EDF from years $t-1$ to t, and $CEDF_t$ is the cumulative EDF for year t.

[28] Specifically, $AEDF_t = 1 - (1-CEDF_t)^{1/t}$, where $AEDF_t$ is the annualized EDF for year t.

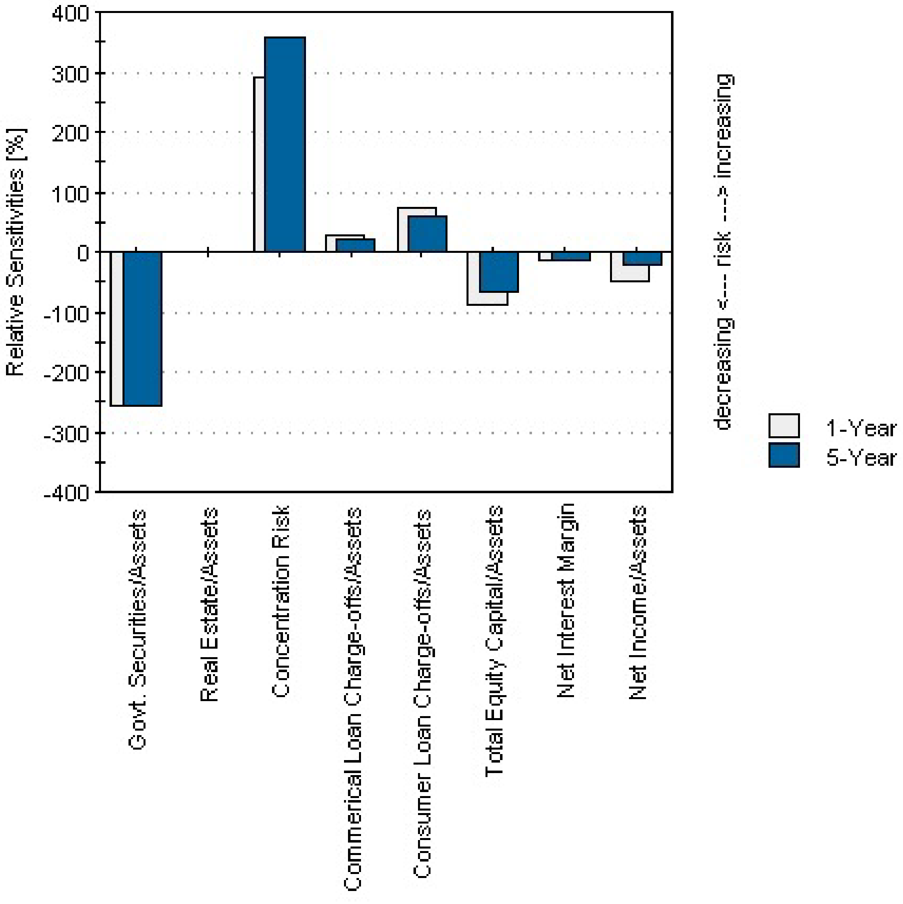
5.2 New Analytical Tools: Relative Sensitivity

The RiskCalc v1.0 interface provides users an analytical tool to gauge the relative impact of each variable—as a deviation from the mean of each ratio. In order to equip the users of the model with further tools, we developed relative sensitivities (also known as sensitivity multiples), which exhibit the EDF sensitivity to each of the model variables at the point of evaluation. This feature is especially useful when addressing the question of identifying variables to improve the EDF credit measure of a bank.

The relative sensitivity gives the impact of a small change in a variable on the EDF credit measure of the bank. It indicates which variables are most sensitive to an increase. A positive number means an increase in the variable will increase risk and a negative number indicates a decrease in risk. The percentile is the sensitivity of the variable relative to the average.

Example: A small increase in Concentration Risk will increase the risk of the company. It is about 300% (1-year) as sensitive as the average variable except government securities/assets at the 1-year level (Figure 14).

FIGURE 16 Relative Sensitivities



6 CONCLUSION

The Moody's KMV RiskCalc v3.1 U.S. Banks model covers all banks, thrifts, and bank holding companies. An additional five years of data was used in the model development, which includes a period of hightented risk in the credit market. We have refined our financial statement model to achieve a robust prediction model of private bank default behavior.

The U.S. Banks v3.1 model is more powerful than its predicisor. We have demonstrated that the increase in power is consistent across bank characteristics groups and size classifications as well as for different time periods. We have also shown that the power advantage is maintained both in-sample and out-of-sample.

In the CCA mode, the U.S. Banks v3.1 model adjusts the EDF credit measure to reflect the current stage of the credit cycle in the banking industry. If default risk in a banking industry is high, the EDF credit measure is adjusted upward. Likewise, when default risk is low, the EDF credit measure is adjusted downward. The CCA adjustment in the U.S. Banks v3.1 model is based upon the Moody's KMV Financial Firm Model for public firms.

Since its release, the U.S. Banks v1.0 model has been widely used for purposes including the risk assesment of colatorlaized debt obligations. The U.S. Banks v3.1 model will be useful for financial institutions for the reasons listed above, plus additional features added to the model. A major advantage of the model is that it provides pre-populated EDF credit measures for private banks based on FDIC and Federal Reserve Bank data. In the U.S. Banks v3.1model, the pre-populated data is updated with quarterly financial statements allowing the user to see the most up-to-date level of risk of the firm. In addition to the pre-populated private firm scores, institutions can use the U.S. Banks v3.1 model as a tool to analyze the risk of publicly traded bank holding companies with EDF credit measures directly from our public financial model.

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